SUMMARY OF PROPERTIES OF CONCAVE GRATINGS IN HOLOGRAPHY

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1. Nature of equiphased surfaces of a system of interferences determined by two monochromatic constant sources.

Thus: C and D are the two source points.

- One supposes the homogeneous middle point of index marked n.

- Given two points A, B one calls the optic path (AB) the quantity n.

AB positive if it is counted going toward the source.

A) - The points C and D are at infinity (Figure 1) or 2θ the angle of their two directions to infinity. The equiphased surfaces are equidistance parallel planes--this is between planes being: $\frac{\lambda}{2n \sin\theta}$

B) - One of the points is at infinity, either C (Figure 2) or (P) a wave plane taken as reference--the points of an equiphase are such that:

(MH) - (MD) = K constant

Let a plane (P') parallel to (P) such as

(MH) = (MH') + (N'H) with (H'H) = K

Then one has for each point of the equiphase K

(MH') - (MD) = o

Let MH' = MD

The equiphases are revolution paraloloids of focus D and of axis parallel to the direction to infinity of C.

C) - C and D are two source points at finite distance.

The equiphases are such that:

(MC) - (MD) = K constant

These are revolution hyperboloids of focus C and D whose axis is a straight line CD.

D) - One of the points, let it be C, is the actual picture perfectly stigmatic from a source point C' (Figure 3).

- One observes the interferences in the portion of space contained between C and the optic projection system (S).

- An equiphase surface is defined by the relation

$$+ (MD) - (MC') = K \text{ constant}$$

but $(MC') = (MC) + (CC')$

and $(CC') = K'$ constant no matter what M is, since C is a perfectly stigmatic picture of C'

Thus one has for an equation of an equiphase:

$$(MD) - (MC) = K + K' = \text{cst.}$$

$$\text{or } n\ (MD + MC) = K + K'$$

$$\text{whence } MD + MC = \text{cst.}$$

The equiphases are thus revolution ellipsoids of focuses C and D whose axis is the straight line CD.

Note:

If D is in C the ellipsoids change into spheres, the equiphases are analogous to the stationary waves which are produced in the neighborhood of a spherical mirror lighted by a constant source placed in its center.

2. The general condition of stigmatism in holographic networks.

- Let $\boxed{C}$ and $\boxed{D}$ be the coherent source points which determine the system of interferences affecting the sensitive surface Σ (Figure 4).

The grooves of the future network correspond to the equiphase intersection line of the volume of interferences by the surface Σ.

- One calls homologous points of a same groove, the points M which have a same order of interferences, thus such as:

(1) $\boxed{MC - MD = O_1 \lambda_0}$ (case of the hyperbolic equiphased surfaces).

- The homologous points of the preceding point on other grooves will be by definition such as:

$$M'C - M'D = (\theta_1 + m)\,\lambda_o \qquad m \text{ entirely } \gtrless 0$$

- Let I be one of these points such as:

(2) $\boxed{IC - ID = \theta_2}$

- The groove to which this point belongs will be conventionally called groove number 0 and by definition $\boxed{\theta_1 - \theta_2 = n}$ (3) will be the number of the groove to which the point M belongs, such as

$$MC - MD = \theta_1\,\lambda_o$$

- Suppose the network completed: one places in $\boxed{A}$ a constant polychromatic source, and one tries to determine the positions of the monochromatic pictures of A diffracted by the network.

- Let $\boxed{B}$ be the picture of A for the wave length λ

- The principle of Fermat permits asserting that B is a perfect picture of A if the optic track MA + MB remains constant no matter what M may be. In the case of the diffracting systems, the quantity MA + MB ought to remain constant the length of a network groove, and only $k\lambda$ ought to vary from one groove to the other (k entirely).

- If the stigmatism is only approximate, the deviation to the stigmatism is characterized by the quantity Δ equal to the variation of the optic track L = MA + MB with respect to an optic track of reference Lθ.

- One can pose:

$$Lo = IA + IB$$

$$L = MA + MB$$

whence $\Delta = L - Lo$ $>$ o if phase retard the length of AMB
$<$ o is phase advance the length of AMB

The principal of Fermat then is written for the diffracted pictures:

$$\boxed{MA + MB = IA + IB + k.n\,\lambda + \Delta(M)} \qquad (4)$$

- k entirely $\gtrless$ being by definition the order of the diffracted picture B of A for the wave length λ

- M being an element of the family of homologous points of groove number n associated with the origin I.

Note:

The whole of the homologous points previously defined constitute a family of lines carried by the miscellaneous grooves of the network.

In order to describe the whole surface of the network, an infinity of such families would be necessary, to which would correspond an infinity of origins I on groove number 0.

The quantity IA + IB which intervenes in the formula (4) therefore depends upon the family of point M in which we are interested; but the variation of this quantity in no way affects the stigmatism of the network, but only its global efficiency.

- The condition of stigmatism remains the condition (4) or Δ would be nil, verified for each family of homologous points taken separately.

- From the equations 1, 2, 3 one can easily take n and carry this value to 4

it happens: $$n = \frac{MC-MD-(IC-ID)}{\lambda_0}$$

whence: $$\boxed{\Delta(M) = MA + MB - (IA + IB) - k\left[MC-MD-(IC-ID)\right]\frac{\lambda}{\lambda_0}}$$

thus: $$\boxed{\Delta(M) = MA+MB - k\frac{\lambda}{\lambda_0}(MC-MD) - \left[IA+IB - k\frac{\lambda}{\lambda_0}(IC-ID)\right]} \qquad (5)$$

Let's pose $$\mathcal{S} = IA + IB - k\frac{\lambda}{\lambda_0}(IC-ID)$$

This is a characteristic constant of the family of homologous points considered.

One has then $\Delta(M) = MA + MB - k \frac{\lambda}{\lambda_o} (MC-MD) - \mathcal{P}$

The condition of stigmatism becomes:

$$\boxed{MA + MB - k \frac{\lambda}{\lambda_o} (MC-MD) = \mathcal{P} = \text{constant}} \qquad (6)$$

- If points A, B, C, and D are distinct, such a relation defines at the most a finite number of curves, but, in our case, the relation ought to be verified by each point M of a quasi-infinity of homologous lines traced on the surface Σ . In order that the problem be possible, it is therefore necessary that the relation decrease in such a way as to cause only two distinct points to intervene.

The relation (6) then becomes a relation of the type

$\lambda \, MP + \mu \, MQ = \text{cst.}$ $\qquad$ λ and μ constant coefficients

It defines then a surface which thus ought to be the surface Σ which carried the network.

- The relation (1) $MC-MD = \theta_1 \lambda_o$ corresponded to the case where the equifaced surfaces are of the hyperbolic type, in the case where these surfaces are ellipsoids, one would have to utilize the relation:

$$(1) \qquad \boxed{MC + MD = \theta_1 \lambda_o}$$

A reasoning analogous to the preceding one would have led to a relation:

$$(5') \qquad \boxed{\Delta(M) = MA+MB - k \frac{\lambda}{\lambda_o} (MC+MD) - \left[IA + IB - k \frac{\lambda}{\lambda_o} (IC+ID)\right]}$$

And to a condition of stigmatism:

$$(6') \qquad \boxed{MA + MB - k \frac{\lambda}{\lambda_o} (MC+MD) = \mathcal{P}' = \text{cst.} \; \text{thus M}}$$

3. <u>Cases of harsh stigmatism for hyperbolic equiphases</u>.

The condition of stigmatism is written:

$$\boxed{MA + MB - k \frac{\lambda}{\lambda_o} (MC-MD) = \rho = \text{cst.}} \qquad (6)$$

- In this case one cannot have C = D

- Miscellaneous cases of the decrease of (6) can be presented:

1.) - A = B = C

One has then: $MA \left(2 - \frac{k\lambda}{\lambda_o}\right) + k \frac{\lambda}{\lambda_o} MD = \rho$

One has harsh stigmatism in autocollimation in A for any given wave length if Σ is the surface of Descartes definited by this relation.

2.) - $\left(\begin{array}{l} A = B = C \\ \lambda = 2 \frac{\lambda_o}{k} \qquad k > o \\ MD = \text{cst.} \end{array} \right.$

$\rightarrow$ MD = cst. implies that Σ is a sphere of center D

[Thus one has harsh stigmatism for $2\frac{\lambda_o}{k}$ in autocollimation on source A when the latter is placed in C.

3.) - Thus A = D

(6) becomes: $MA \left(1 + \frac{k\lambda}{\lambda_o}\right) + MB - \frac{k\lambda}{\lambda_o} MC = \rho$

-a) - if in addition $\lambda = - \frac{\lambda_o}{k} \rightarrow k < o$

it happens that $MB + MC = \rho$

$\rightarrow$ Σ ought to be an ellipsoid of focus B and C.

If surface Σ is an ellipsoid of focus B and C, C being one of the registration points, there is harsh stigmatism in B for the wave length $-\frac{\lambda_0}{k}$ in the negative orders when one places the source in D.

- b) - If, in addition MA = cst. with B = C

Σ is therefore a sphere of center A, D

One observes at point C

There is harsh stigmatism in C for $\frac{\lambda_0}{k}$ $k > 0$

- c) - One always has $\begin{cases} A = D \\ MA = \text{cst.} \end{cases}$

A and D are at the center of a spherical cap if in addition one has

$$\boxed{(7) \qquad Mg - \frac{k\lambda}{\lambda_0} MC = 0 \qquad \text{qq.} \quad \text{thus } \Omega}$$

One has in B a perfectly stigmatic picture for the wave length λ whatever given

Let's propose $\boxed{\lambda = m \cdot \frac{\lambda_0}{k}}$ m

Relation 7 becomes:

$\boxed{MB - m\, MC = 0}$ whatever may be M

It means (Figure 5) that points B and C ought to be situated on a diameter P, Q of the sphere Σ and that P and Q ought to divide harmonically the segment BC in the relation m

$$\frac{PB}{PC} = \frac{QB}{QC} = m.$$

It follows that one also has the relation:

$$\boxed{\frac{DP}{DC} = \frac{DB}{DP} = m} \Rightarrow \boxed{DC = \frac{R}{m}} \qquad \boxed{DB = mR}$$

Relations which permit determining the position of C and that of the point of observation B.

In summary, if Σ is a sphere of center D (MD = cst.) and if one considers B conjugated harmonic of point C with relation to a diameter of the sphere: (MB = m MC)

- When source A is at the center: MA = MD = cst.

- There is harsh stigmatism:

  . in C for the wave length $\frac{\lambda_o}{k}$

  . in B for the wave length $m\,\frac{\lambda_o}{k}$

- The stigmatism expressing the fact that optic tracks are stationary, one ought to obtain a stigmatism for other wave lengths when, while utilizing the same network, one places the source in C or B and one observes in one of the points D, C or B.

- Let's analyze the different cases:

- The hypotheses are:

  spherical surface of center D: MD = cst. (
  (
  Point $\boxed{R}$ conjugated harmonic of C (See Figure 6
  (
  with relation to a diameter: MR = m MC (

1^{o}) - Let's place the source in C: MA =MC (
(
One observes in R for what wave length there (
(
is stigmatism: MB = MR (

The condition (6) $\boxed{MA + MB - k\,\frac{\lambda}{\lambda_o}\,(MC-MD) = \mathcal{S} = cst.}$

becomes: $MC + MR - k\,\frac{\lambda}{\lambda_o}\,(MC-MD) = \mathcal{S} = cst.$

thus: $MC\,(1 + m - k\,\frac{\lambda}{\lambda_o}) + k\,\frac{\lambda}{\lambda_o}\,MD = \mathcal{S} = cst.$

This condition is completed if one nullifies the term in MC

Thus for $\boxed{\lambda = (m + 1)\,\frac{\lambda_o}{k}}$ $\quad k > o$

- One has, one the other hand, already shown that for the same position of the source there is stigmatism

in D for $\frac{\lambda_o}{k}$

in C for $\frac{2\lambda_o}{k}$

2^o) - Let's place the source in R: MA = MR (
(
a) - Let's observe in D: MB = MD (

The condition (6) becomes:

$$MR + MD - k\,\frac{\lambda}{\lambda_o}\,(MC - MD) = \rho = cst.$$

Thus:

$$MC\,(m - k\,\frac{\lambda}{\lambda_o}) + MD\,(1 + k\,\frac{\lambda}{\lambda_o}) = \rho \quad cst.$$

The condition is satisfactory for $\boxed{\lambda = m\,\frac{\lambda_o}{k}}$ $\quad k > o$

b) - Source in R: MA = MR (
(
One observes in C: MB = MC (

The condition (6) becomes:

$$MR + MC - k\,\frac{\lambda}{\lambda_o}\,(MC - MD) = \rho = cst.$$

$$MC\,(1 + m - k\,\frac{\lambda}{\lambda_o}) + k\,\frac{\lambda}{\lambda_o}\,MD = \quad = cst.$$

There is stigmatism for $\boxed{\lambda = (m+1)\,\frac{\lambda_o}{k}}$ $\quad k > o$

c) - Source in R: MA = MB (
(
One observes in R: MB = MR (

The condition (6) becomes:

$$2\,MR - k\,\frac{\lambda}{\lambda_o}\,(MC - MD) = \quad = \rho = cst.$$

$$\text{Thus } MC\,(2m - k\,\frac{\lambda}{\lambda_o}) + k\,\frac{\lambda}{\lambda_o}\,MD = \rho = cst.$$

There is stigmatism for $\boxed{\lambda = 2\,m\,\frac{\lambda_o}{k}}$ $\quad k > o$

In summary: - In the case where the sensitive support is a sphere and where one of the registration points is at its center, there is thus, in general, three points of harsh stigmatism:

- the two registration points

- the point conjugated harmonic of the registration point which is not at the center of the network with regard to the diameter of the network.

- the following schemas summarize the properties of such networks:

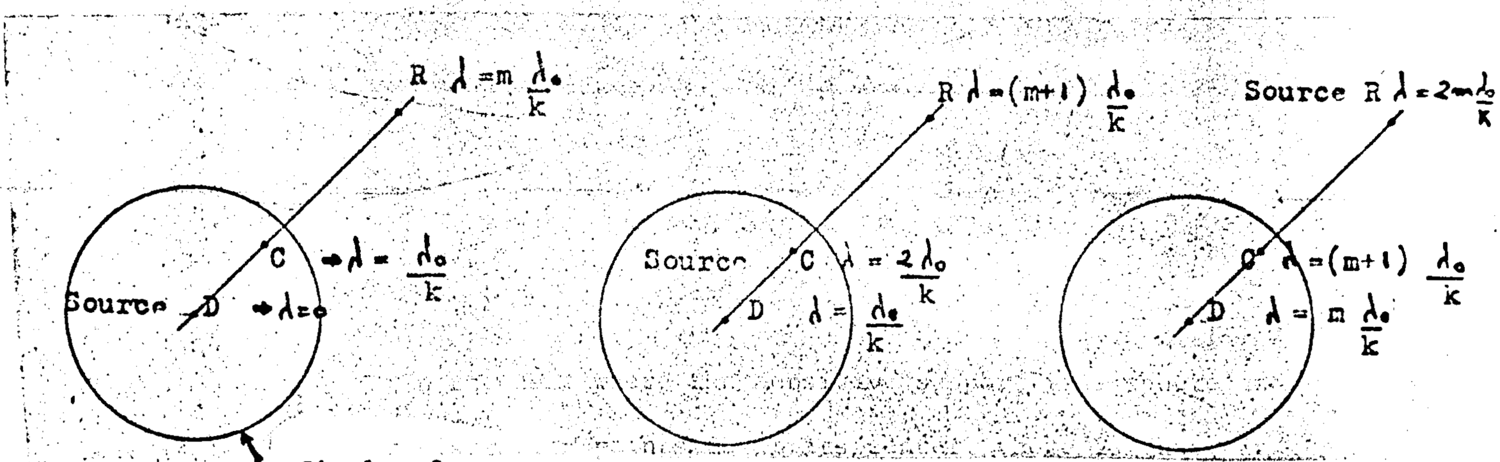
$\frac{MR}{MC} = m$	$DC = \frac{R}{m}$	$DR = m.r.$

One has: $\frac{MR}{MC} = m$ $DC = \frac{R}{m}$ $DR = m.r.$

- a) - If $m > 1$

- One has necessarily $m < \frac{1}{n.\lambda o}$ n = Number of lines/mm of the network measured in the neighborhood of the summit.

- C is at the interior of the circle of the network

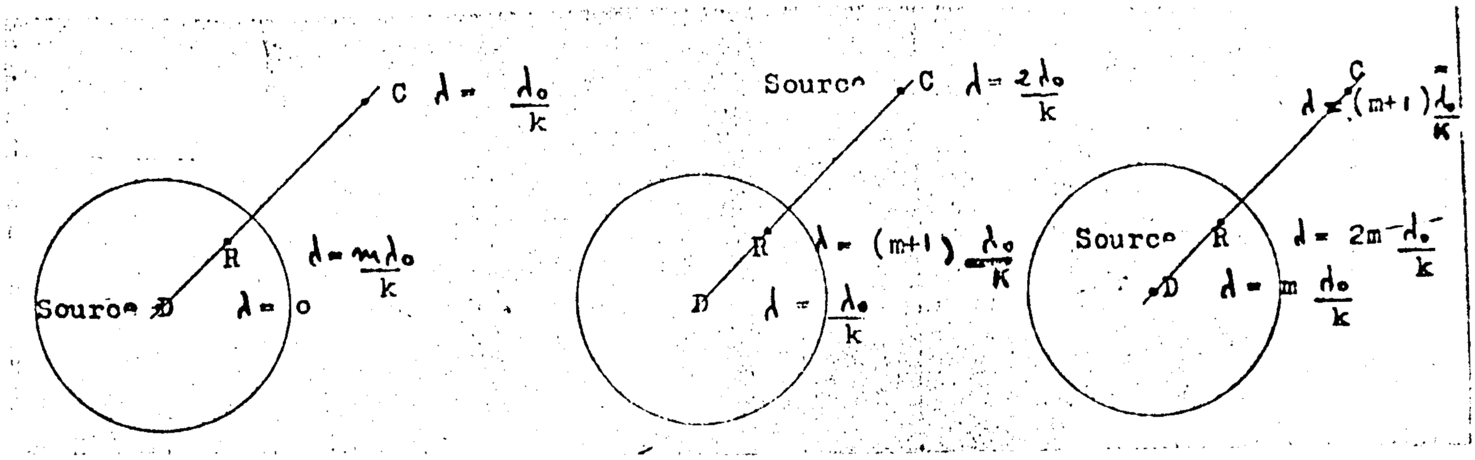


Circle of the network

- b) - If $m < 1$

- No limitation on m

- C is at the exterior of the circle of the network.



- Figure 7 indicates the zone of the plane where is found:

- Point C according to the values of m.

- If C penetrates the interior of the circle of Rowland, the third point of stigmatism becomes potential, whence a lesser interest in the configuration.

4°/ - Cases of harsh stigmatism for elliptical equiphases.

- The condition of harsh stigmatism is written then: (6')

$$\boxed{MA + MB - k\frac{\lambda}{\lambda_o}(MC + MD) = \rho = cst.} \qquad (6')$$

In this case one can have C = D (spherical equiphases)

- One can write (6') in the form:

$$\left(MA - k\frac{\lambda}{\lambda_o}MC\right) + \left(MB - k\frac{\lambda}{\lambda_o}MD\right) = \rho = cst.$$

It is then obvious that for:

(A = C → source point in C

(B = D → observation point in D

($\lambda = \frac{\lambda_o}{k}$ $\quad k > o$

The relation (6') is satisfied:

One has harsh stigmatism no matter what the form of surface Σ for the wave length $\frac{\lambda_o}{k}$, when one places the source in one of the registration points, and one observes at the other.

- All the cases of stigmatism obtained in the preceding paragraph are equally valid in the case of elliptical equiphases, particularly when the support is spherical and when one of the registration points is located at the center of the network.

- New possibilities are however permitted by the fact that one can have C and D mixed:

thus $C = D$

The conditions (6') becomes:

$$\boxed{MA + MB - 2k\frac{\lambda}{\lambda_0}MC = \rho = \text{cst.}}$$

The case $MD = \text{cst.}$ being excluded, one is led to consider the case:

$MA = \text{cst.}$

Which implies a spherical support with the source point at the center of the network: it happens that:

$$MA + \left(MB - 2k\frac{\lambda}{\lambda_0}MC\right) = \rho - \text{cst.}$$

let's propose $m = 2k\dfrac{\lambda}{\lambda_0}$

The condition of stigmatism is thus satisfied for whatever wave length:

$$\boxed{\lambda = m\frac{\lambda_0}{2k}} \qquad k > 0 \qquad \text{if one has in addition:}$$

$$(7) \quad \boxed{MB - m\ MC = 0}$$

This implies that B is in R conjugated harmonic of registration point C with regard to a diameter PQ of the sphere of the network Figure 8.

Which implies the relation:

$$\frac{PB}{PC} = \frac{QB}{QC} = m = \frac{PR}{PC} = \frac{QR}{QC}$$

Or still if O is the center of the network:

$$\frac{OP}{OC} = \frac{OR}{OP} = m$$

Thus
$$\boxed{\begin{aligned} OC &= \frac{R}{m} \\ OR &= m.r. \end{aligned}}$$
R. radius of curvature of the network

- One can also satisfy the condition (7)

by making (B = C

and (m = 1

Therefore one has harsh stigmatism for the wave length $\frac{\lambda_o}{2k}$ if one places the source at the center of the network and one observes at the common point of registration.

- As in the preceding paragraph, one can expect other conditions of stigmatism when one utilizes the same network with the source A and C, D or at point R conjugated of C, D.

- The hypotheses are therefore:

- Spherical support of center : 0
- Mixed registration points : MC = MD
- R conjugated harmonic of C, D with regard to a network diameter : MR = m. MC

- 1^o/- One places the source A in C, D : MA = MC = MD

One observes at the center of the network: MB = cst.

The condition (6') becomes:

$$MC \left(1 - 2k \frac{\lambda}{\lambda_o} \right) + MB = \rho = cst.$$

It is therefore satisfied for the wave length

$$\boxed{\lambda = \frac{\lambda_o}{2k}} \qquad k > o$$

-2^o/- One places source A in C, D : MA = MC = MD

One observes in C, D MB = MC

- One has seen that there is stigmatism for $\boxed{\lambda = \frac{\lambda_o}{k}}$ $k > o$

no matter what the form of the support may be.

-3°/- Source A in C, D : MA = MC = MD

One observes at point R : MB = MR

The condition of stigmatism becomes:

$$MC\ (1-2k\ \frac{\lambda}{\lambda_o})\ +\ MR = \rho = cst.$$

$$MC\ (1-2k\frac{\lambda}{\lambda_o}\ +\ m) = \rho = \quad cst. =$$

It is therefore satisfied for the wave length

$$\boxed{\lambda = (m + 1\ )\ \frac{\lambda_o}{2\ k}} \qquad k > o$$

-4°/- One places the source in R : MA = MR

One observes at the center of the network: MB = cst.

The condition (6') becomes:

$$MR + MR - 2\ k\ \frac{\lambda}{\lambda_o}\ MC = \rho = cst.$$

thus $$MB + (\ m - 2\ k\ \frac{\lambda}{\lambda_o})\ \ MC = \rho = cst.$$

The condition is satisfied for:

$$\boxed{\lambda = m\ \frac{\lambda_o}{2\ k}}$$

-5°/- One places the source in R : MA = MR

One observes in C, D : MB = MC

The condition (6') becomes:

$$MR + MC\ (1-\ 2k\ \frac{\lambda_o}{\lambda_o}) = \rho = cst.$$

thus $$MC\ (m + 1 = 2\ k\ \frac{\lambda}{\lambda_o}\ =)\ \rho\ = cst.$$

The condition is satisfied for the wave length

$$\boxed{\lambda = (m + 1)\ \frac{\lambda_o}{2\ k}} \qquad k > o$$

-6°/- One places the source in R : MA = MR

One observes in R MB = MR

The condition (6') becomes:

$$2\ MR\ -\ 2k\ \frac{\lambda}{\lambda_o}\ \ MC = \rho\ = cst.$$

$$MC\ (2m\ -\ 2k\ \frac{\lambda}{\lambda_o})\ = \rho = cst.$$

There is harsh stigmatism for the wave length

$$\boxed{\lambda = \frac{m\lambda_o}{k}} \quad k > o$$

One can summarize by schemas the preceding properties:

- Holographic networks with spherical support of center 0, radius R.
- Equiphase surfaces of elliptical type - registration points C, D mixed.
- Point R conjugated harmonics of C, D with regard to a diameter of the network:

$\frac{MR}{MC} = m \quad OC = \frac{R}{m} \qquad OR$

a)- If $> m \quad 1$

One has necessarily m — n = number of lines/mm in the neighborhood of the summit; λo = impression wave length

C, D are at the interior of the circle of the network.

- Holographic networks with spherical support of center 0, radius R.
- Equiphased surfaces of elliptical type--registration points C, D mixed.
- Point R conjugated harmonics of C, D with regard to a diameter of the networks:

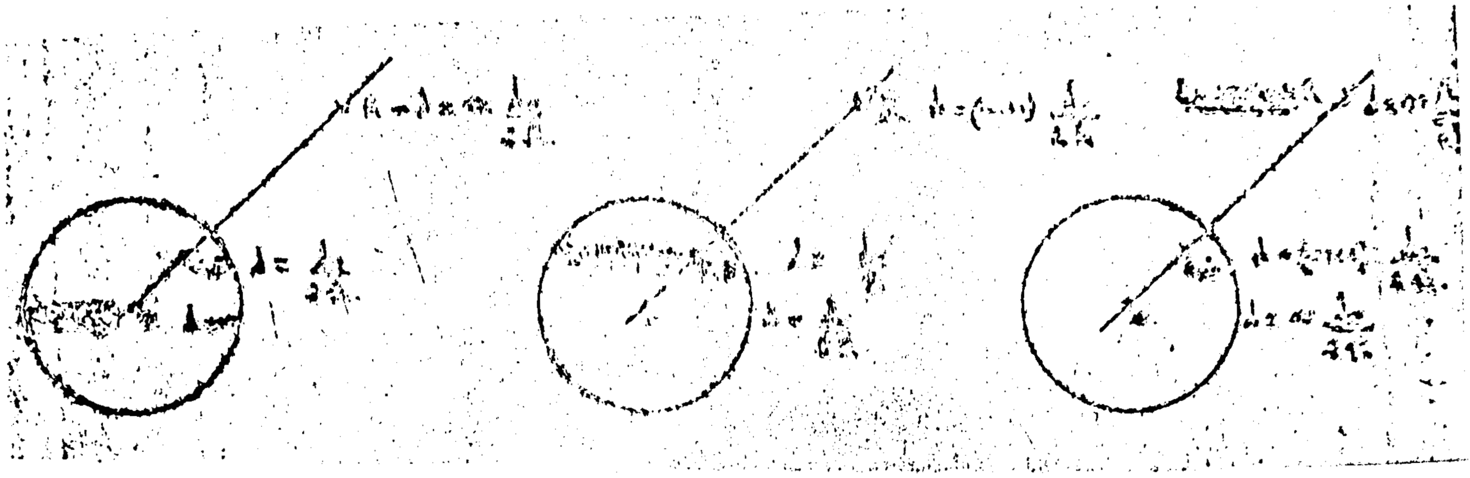
$\frac{MR}{MC} = m \qquad OC = \frac{R}{m} \qquad OR = m.r.$

(above equation may be incorrect)

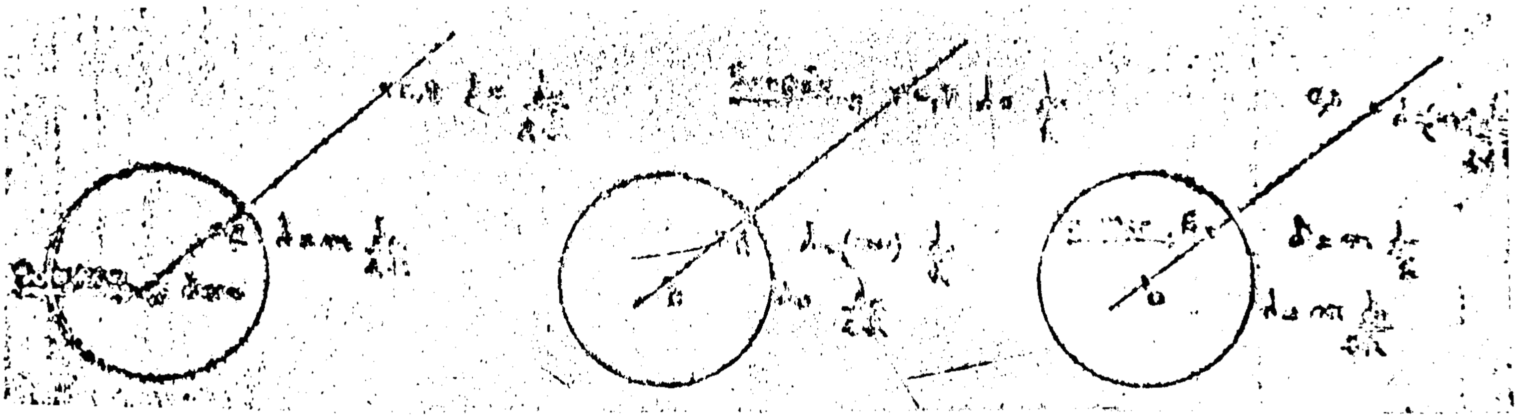
If $m > 1$

- One has necessarily (unreadable) n = number of lines/mm is the neighborhood of the summit; (unreadable) = impression wave length

- C, D are at the interior of the circle of the network.



If $m < 1 \rightarrow$ no limitations on m



(unreadable) indicates the zones of the plane where are found points (unreadable) indicating the values (unreadable)

- <u>Networks by holography</u>

<u>Use of a convex support</u>

One always supposes that one of the registration waves is centered on the network--in our case it is a convergent wave with B its center.

-A)- The other registration wave is also convergent with C its center (Figure 1).

The points C and D are the potential pictures of two coherent source points S_c and S_d.

- Points M of a same equiphase line at the surface of the network are such that:

$$S_c\, M - S_d\, M = \theta_1\, \lambda_o$$

or I the point of origin

$$S_c I - S_d I = \theta_2\, \lambda_o$$

- The number of the groove is by definition: $\theta_1 - \theta_2$

thus: $$n = \frac{1}{\lambda_o} \left[(S_c M - S_d\, M) - (S_c I - S_d\, I) \right]$$

But, one can write:

$$S_c M = K_c - MC \text{ with } K_c = (S_c\, C)$$

and $$S\, M = K_c - MC \text{ and } K = (S,\, D)$$

whence $$\boxed{n = \frac{1}{\lambda_o} \left[MD - MC - (ID - IC) \right]} \qquad (1)$$

-1°/ Let's suppose the network completed and let's look again for the real pictures B diffracted by the network lighted by a convergent polychromatic wave centered in D (D potential picture from a real source S_d)

If a stigmatic picture of A exists for one ought to have:

$$S_d\, M + MB = S_d\, I + IB + nk\,\lambda \qquad \text{(k on the order } \pm\text{)}$$

thus $$MB - MD = IB - ID = nk\,\lambda$$

Let's replace n by its value (1) it happens that:

$$\boxed{MB - MD - k\,\frac{\lambda}{\lambda_o}\,(MD-MC) = IB - ID - k\,\frac{\lambda}{\lambda_o}\,(ID-IC)}$$

but $$MD = cst.$$

The condition of stigmatism therefore becomes:

$$MB + k\,\frac{\lambda}{\lambda_o}\,MC = cst.$$

The condition is thus filled for

$$\begin{cases} k = -1 \quad \lambda = \lambda_o & \text{if } B = C \\ k = -1 \quad \lambda & \text{if } \dfrac{MB}{MC} = -k\dfrac{\lambda}{\lambda_o} \end{cases}$$

- The first solution is not physically possible

Point C being by construction, potential

- The last solution is possible for a fraction of the surface of the network (see Figure 2) if B and C are conjugated harmonic with regard to a network diameter in the relation $m = -k\dfrac{\lambda}{\lambda_o}$

One has then: $\begin{cases} DB = m\,R \\ DC = \dfrac{R}{m} \end{cases}$ R radius of the network.

-2°/ Let's look again for the potential pictures B diffracted by the network lighted by a convergent wave centered in D.

One ought to have:

$$S_d\,M + M\,\Sigma_B = S_d\,I + I\,\Sigma_B + n\,k\,\lambda$$

Σ_B being a wave centered on the eventual picture B

But one has $S_D\,M = K_D - MD \qquad K_D = (S_D\,D)$

and $M\Sigma_B = K_B - MB \qquad K_B = (B\Sigma_B)$

whence

$$\boxed{MB + MD = ID + IB - n\,k\,\lambda}$$

By replacing n by its value the condition of stigmatism becomes

$$\boxed{MB + MD + k\frac{\lambda}{\lambda_o}(MD - MC) = IB + ID + k\frac{\lambda}{\lambda_o}(ID - IC)}$$

It is satisfied for

$k = -1 \quad \lambda = \lambda_o \qquad \text{if } B = C$

$k = = 1 \quad \lambda \qquad \text{if } MB + k\dfrac{\lambda}{\lambda_o}MC = o$

- The first solution is valid.

- The second is only physically possible for a fraction of the surface of the network (see Figure 4) if B and C are conjugated harmonics in the relation $m = -k \frac{\lambda}{\lambda_o}$ with regard to a diameter of the network.

-5°)- <u>Diffraction by a holographic network of a wave of given form</u>.

- Let's take up again the demonstration of 2^o

If one distorts the waves emitted by points C and D at the registration by modifying the optic tracks (MC) and (MD) of well determined quantities $\delta c(M)$ and $\delta D(M)$ by interposing a phase scale on the path of these radii, the equations (1) and (2) become:

$$(MC + \delta c(M)) - (MD + \delta_D (M)) = \theta_1 \lambda o$$
$$(IC + \delta c(I)) - (ID + \delta_D (I)) = \theta_2 \lambda o$$

The expression of $n = \theta_1 - \theta_2$ becomes

$$n = \frac{MC-MD - (IC-ID) + \delta c(M) - \delta_D(M) - (\delta c\ (I) - \delta_D\ (I))\ s_D\ (I))}{\lambda o}$$

The expression of the aberrant deviation Δ (M) relative to the wave diffracted by the network for the wave length λ becomes:

$$\Delta (M) = MA+MB - (IA+IB) -k \frac{\lambda}{\lambda o} \left[MC-MD-(IC-ID)\right] -k\frac{\lambda}{\lambda o}\left[\delta c(M) - \delta_D(M) - (\delta_c(I) - \delta_D(I))\right]$$

By considering the deformation δ beginning with principle radii, one can cause $\delta c\ (I) = \delta_D\ (I) = o$

whence by introducing the constant : ρ

$$\boxed{\Delta(M) = MA+MB-k \frac{\lambda}{\lambda o} (MC-MD) - \rho - k \frac{\lambda}{\lambda o} (\delta c (M) - \delta_D (M))}$$

If one distorts only one of the waves at registration:

$$\boxed{\text{One has} \quad \Delta(M) = MA + MB - k \frac{\lambda}{\lambda o} (MC-MD) - \rho + k \frac{\lambda}{\lambda o} \delta_D\ (M)}$$

One can therefore distort at will the diffracted waves, but it must be noticed that:

- the sign of the deviation introduced reverses itself accordingly as one utilizes the order + or -

- the value of the deviation introduced varies linearly with the wave length.

-6°/- <u>The aberrations of the 3° order of the holographic networks</u>.

a) <u>Generalities</u>.

- Outside the points of harsh stigmatism, the expression of aberrant deviation

$$\boxed{\Delta(M) = MA + MB - K\frac{\lambda}{\lambda_0}(MC \pm MD) - \mathcal{G}} \qquad (5)$$

is different from zero, the network diffracts a wave blemished with aberrations.

- One will try to obtain an approximate value of the aberrant deviation by making a limited development of the expression of Δ (5)

- In the case of the diffraction networks the angles between beams, object and picture are in general important--the field variables will not therefore be taken as infinitely small in the first order, the limited development of Δ(M) will be made with regard to the only variables of opening.

- Let α' and Ψ be the pupillary coordinates in the picture space (see Figure 9), one will put Δ in the form:

$$\Delta = \delta_1 . \alpha' + \delta_a . \alpha'^2 + \delta_c \alpha'^3 + \delta_s \alpha'^4 + \varepsilon(\alpha'^5) \ldots .$$

The quantities δ depending on the variable Ψ of the coordinates of the source point and of observation.

- δ_a will be the equivalent of the term of astigmatism, δ_c of the term of Coma and δ_s of the term of spherical aberration. The coefficient δ_1 of the term of the first degree will be capable of being nullified, thus furnishing a relation between the directions of the source and of its picture.

- One will be able to continue to the transversal deviations by utilizing the relations of Hamilton:

$$\begin{cases} n\,dy = \dfrac{\cos\varphi}{\cos\alpha'} \cdot \dfrac{\partial \Delta}{\partial \alpha'} - \dfrac{\sin\varphi}{\sin\alpha'} \cdot \dfrac{\partial \Delta}{\partial \varphi} \\[2ex] n\,dz = \dfrac{\sin\varphi}{\sin\alpha'} \cdot \dfrac{\partial \Delta}{\partial \alpha'} + \dfrac{\cos\varphi}{\sin\alpha'} \cdot \dfrac{\partial \Delta}{\partial \varphi}. \end{cases}$$

- b) - <u>The limits of validity of limited development</u>.

Let's mark the space in relation to 3 rectangular axes of common origin, I, the summit of the network (see Figure 10).

- Let A be the source point which illuminates the network.

- The observation point B will be chosen on the radius diffracted by the summit I of the network for the radiation λ

Let B' be the projection of point B on the radius diffracted by a point M of the network for the same wave length, the quantity BB' is on the same order of size as the transversal deviation.

The aberrant deviation is equal to the quantity:

$$\Delta_a = (AMB') - (AIB) - P\lambda \qquad \text{P entirely}$$

but we calculate the quantity:

$$\Delta = (AMB) - (AIB) - P\lambda$$

We therefore commit a systematic error on the order of the quantity $(BB')^2$ or on the order of the deviation transversal to the square.

- In the hypothesis where one could nullify the terms of the first and the second degree in α', one would again commit an error of the fourth order in α' on the value of the aberrant deviation.

- Given that one cannot, in general, completely nullify the terms of astigmatism, it is useless to push the limits of development above the third order.

However, in the neighborhood of the points of harsh stigmatism, the terms of the fourth degree can be taken into consideration. Their expression being extremely heavy, we shall limit ourselves to the terms of Coma.

c) - <u>The different terms of aberration</u>:

- The space is marked in relation to the three rectangular axes $\vec{IX}$, $\vec{IY}$, $\vec{IZ}$ of origin I, the summit of the network (Figure 10).

The equation of the surface which carries this last is then

$$\nu X^2 + \lambda Y^2 + \mu Z^2 - 2RX = 0$$

$\lambda = \mu = \nu = 1$: sphere

$\nu = 0$ paraloloid

- A point of the space is marked by three coordinates x, y, z and on proposes

$$l^2 = x^2 + y^2 + z^2$$

- To carry out the limits of development to the third order of $\Delta = MA + MB - k\frac{\lambda}{\lambda_0}(MC \pm MD) - \Omega$ it necessary to obtain the development in the same order of the quantities MA, MB, MC, MD which regard to Y and Z.

One has: $MA^2 = (X-x)^2 + (Y-y)^2 + (Z-z)^2$

thus $MA^2 = x^2 + y + z^2 - 2(yY - zZ) - 2\,xX + Y^2 + Z^2 + X^2$

From the equation of the surface of the network one takes:

$$X = \frac{\lambda Y^2 + \mu Z^2}{2R} + \nu \frac{(\lambda Y^2 + \mu Z^2)^2}{8R^3} + \mathcal{E}(Y^6, Z^6)$$

$$X^2 = \frac{(\lambda Y^2 + \mu Z^2)^2}{4R^2} + \mathcal{E}(Y^6, Z^6)$$

By carrying these values in the expression of MA^2, it happens that:

$$MA^2 = \ell^2\left[1 - 2\frac{(yY + zZ)}{\ell^2} + \frac{Y^2 + Z^2}{\ell^2} - \frac{x}{R\ell^2}(\lambda Y^2 + \mu Z^2) + \frac{(\lambda Y^2 + \mu Z^2)^2}{\ell^2}\cdot\left(\frac{1}{4R^2} - \frac{\nu x}{4R^3}\right) + \right]$$

thus
$$MA^2 = \ell^2(1 + \theta)$$

therefore
$$MA = \ell\left(1 + \frac{\theta}{2} - \frac{\theta^2}{8} + \frac{\theta^3}{16} + \dots\right)$$

with

$$\theta = \frac{1}{\ell^2}\left[-2(yY + zZ) + Y^2 + Z^2 - \frac{x}{R}(\lambda Y^2 + \mu Z^2) + (\lambda Y^2 + \mu Z^2)^2\left(\frac{1}{4R^2} - \frac{\nu x}{4R^3}\right)\right]$$

By ordering (unreadable)

$$MA = \left[\begin{array}{l} \ell \\ - \dfrac{yY + zZ}{\ell} \\ + \dfrac{1}{2\ell}\left[Y^2\left(1 - \dfrac{\lambda x}{R} - \dfrac{y^2}{\ell^2}\right) + Z^2\left(1 - \dfrac{\mu x}{R} - \dfrac{z^2}{\ell^2}\right) - 2\dfrac{yzYZ}{\ell^2}\right] \\ + \dfrac{yY + zZ}{2\ell^3}\left[Y^2\left(1 - \dfrac{\lambda x}{R} - \dfrac{y^2}{\ell^2}\right) + Z^2\left(1 - \dfrac{\mu x}{R} - \dfrac{z^2}{\ell^2}\right) - 2\dfrac{yzYZ}{\ell^2}\right] \\ \\ + \text{terms of the fourth degree} \end{array}\right.$$

- In the first time one will be interested in points located in the plane X Y, therefore such as

$$z = o$$

- If α is the oriented angle from the vector radius $\overrightarrow{IA}$ with the axis $\overrightarrow{IX}$ one has on the other hand:

$$\left(\begin{array}{l} x = l \cos \alpha \\ \\ y = l \sin \alpha \end{array} \right.$$

- The expression of M A is then:

$$
\begin{array}{ll}
MA = & \left[\begin{array}{l} \ell \\
\text{terms of the 1st order} \rightarrow & - Y \sin \alpha \\
\text{terms of the 2nd order} \rightarrow & + \frac{Y^2}{2} \left(\frac{\cos^2 \alpha}{\ell} - \frac{\mu \cos \alpha}{R} \right) + \frac{Z^2}{2} \left(\frac{1}{\ell} - \frac{\mu \cos \alpha}{R} \right) \\
\text{terms of the 3rd order} \rightarrow & + \frac{Y^3}{2} \frac{\sin \alpha}{\ell} \left(\frac{\cos^2 \alpha}{\ell} - \frac{\mu \cos \alpha}{R} \right) + \frac{Y Z^2}{2} \frac{\sin \alpha}{\ell} \left(\frac{1}{\ell} - \frac{\mu \cos \alpha}{R} \right) \\
\text{terms of the 4th order and the following} \rightarrow & + Y^4 \ldots\ldots\ldots
\end{array} \right.
\end{array}
$$

- Now let's replace in the expression of the aberrant deviation

$$\left[\begin{array}{ll} & \Delta = MA + MB - k \frac{\lambda}{\lambda_0} (MC \pm MD) - \rho \\ \text{with} & \rho = IA + IB - k \frac{\lambda}{\lambda_0} (IC + ID) \end{array} \right.$$

The different terms MA, MB, MC, MD by their development

We shall characterize the points A, B, C, D by polar coordinates:

Source points	A :	α	l_A	= IA	
Observation point	B :	β	l_B	= IB	
1st registration point	C :	γ	l_C	= IC	(Figure 11)
2nd registration point	D :	δ	l_D	= ID	

It happens that:

$$
\begin{aligned}
\Delta \; = \;& -Y\left[\sin\alpha + \sin\beta - k\frac{\lambda}{\lambda_0}\left(\sin\gamma \pm \sin\delta\right)\right] \\
\Delta_T \rightarrow \;& + \frac{Y^2}{2}\left[\frac{\cos^2\alpha}{\ell_A} - \frac{\lambda\cos\alpha}{R} + \frac{\cos^2\beta}{\ell_B} - \frac{\lambda\cos\beta}{R} - k\frac{\lambda}{\lambda_0}\left(\frac{\cos^2\gamma}{\ell_C} - \frac{\lambda\cos\gamma}{R}\right) \mp k\frac{\lambda}{\lambda_0}\left(\frac{\cos^2\delta}{\ell_D} - \frac{\lambda\cos\delta}{R}\right)\right] \\
\Delta_S \rightarrow \;& + \frac{Z^2}{2}\left[\frac{1}{\ell_A} - \mu\frac{\cos\alpha}{R} + \frac{1}{\ell_B} - \frac{\mu\cos\beta}{R} - k\frac{\lambda}{\lambda_0}\left(\frac{1}{\ell_C} - \frac{\mu\cos\gamma}{R}\right) \mp k\frac{\lambda}{\lambda_0}\left(\frac{1}{\ell_D} - \frac{\mu\cos\delta}{R}\right)\right] \\
\Delta_C \rightarrow \;& + \frac{Y^3}{2}\left[\frac{\sin\alpha}{\ell_A}\left(\frac{\cos^2\alpha}{\ell_A} - \frac{\lambda\cos\alpha}{R}\right) + \frac{\sin\beta}{\ell_B}\left(\frac{\cos^2\beta}{\ell_B} - \frac{\cos\beta \cdot \lambda}{R}\right) - k\frac{\lambda}{\lambda_0}\frac{\sin\gamma}{\ell_C}\left(\frac{\cos^2\gamma}{\ell_C} - \frac{\lambda\cos\gamma}{R}\right) \mp k\frac{\lambda}{\lambda_0}\frac{\sin\delta}{\ell_D}\left(\frac{\cos^2\delta}{\ell_D} \frac{\lambda\cos\delta}{R}\right.\right. \\
& + \frac{YZ^2}{2}\left[\frac{\sin\alpha}{\ell_A}\left(\frac{1}{\ell_A} - \frac{\mu\cos\alpha}{R}\right) + \frac{\sin\beta}{\ell_B}\left(\frac{1}{\ell_B} - \frac{\mu\cos\beta}{R}\right) - k\frac{\lambda}{\lambda_0}\frac{\sin\gamma}{\ell_C}\left(\frac{1}{\ell_C} - \frac{\mu\cos\gamma}{R}\right) \mp k\frac{\lambda}{\lambda_0}\frac{\sin\delta}{\ell_D}\left(\frac{1}{\ell_D} - \frac{\mu\cos\delta}{R}\right)\right]
\end{aligned}
$$

+ 4th order

The sign $\pm$ in the last term of each parentheses correspond to the case of the ellipsoidal equiphase surfaces (sign $-$) and hyperbolic equiphases (sign +).

- In order that the limited development have a direction, we have seen that the terms of the first degree at least ought to be nil, one ought therefore always to have the relation:

$$\boxed{\sin\alpha + \sin\beta - k\frac{\lambda}{\lambda_0}\left(\sin\gamma \pm \sin\delta\right) = 0} \qquad (8)$$

Thus in proposing:

$$a = \frac{\lambda_0}{\sin\gamma \pm \sin\delta}$$

$$\boxed{a\left(\sin\alpha + \sin\beta\right) = k\lambda}$$

This is the standard formula of the networks:

The radii IA and IB ought therefore to correspond by diffraction on a network of step a in the order k.

- This condition can be physically accomplished in the case which interests us, for the quantity $a = \dfrac{\lambda_o}{\sin\gamma \pm \sin\delta}$ represents precisely the step of the interference fringes which are produced in the neighborhood of the summit I of the surface Σ lighted with the wave lengths λ_o. by the two coherent points C and D of polar angle γ and δ.

- Among the terms of astigmatism and Coma, these are the terms in Y^2 and Y^3 which are the most bothersome: these are the ones, indeed, which give an enlargement of the picture perpendicular to the lines of the network.

- Thus, taking into account the relation (8), the nullification of the coefficient of Y^2 gives the polar location of the tangential focal:

$\ell_T = F(\beta)$

- In like manner, the nullification of the coefficient of Z^2 would give the polar location of the sagittal focal: $\ell_S = G(\beta)$

- And that of a coefficient of Y^3 the polar location of least Coma.

$\ell_C = H(\beta)$.

7°/- Study of particular cases

1°/- The study of the conditions of harsh stigmatism has shown us that in the case where the support of the network is spherical, and where one of the registration points is at its center, three points of perfect stigmatism exist.

- We are going to study in this particular case the form of the focal curves.

Before making the study of these curves, we are going to show the following proposition:

The focal curves, sagittal and tangential, and the curve of no Coma are the same, respectively, as the source point, whether it be at the center of the network, at point C, or at point R conjugated from point C - see (Figure 6).

- We are going to show this proposition in the case of the tangential focal curve.

Like demonstrations could be given for the location of the sagittal focal and no coma.

- a) - Source point A is at the center of the network $\rightarrow \alpha = 0 \quad \ell_A = R$

Point D is also at the center $\rightarrow \delta = 0 \quad \ell_D = R$

- By carrying these values in the term in Y^2 of the general expression of Δ it happens that:

$$\frac{\cos^2\beta}{\ell_\Delta} - \frac{\cos\beta}{R} - k\frac{1}{\lambda_0}\left(\frac{\cos^2\gamma}{\ell_C} - \frac{\cos\gamma}{R}\right) = 0$$

with $$\sin\beta - k\frac{1}{\lambda_0}\sin\gamma = 0$$

The polar equation of the tangential focal curve is thus given by the relation:

$$\frac{\cos^2\beta}{\ell_T} - \frac{\cos\beta}{R} - \frac{\sin\beta}{\sin\gamma}\left(\frac{\cos^2\gamma}{\ell_C} - \frac{\cos\gamma}{R}\right) = 0$$

- b) - Let's put source point A in C: $\alpha = \gamma \quad \ell_A = \ell_C$

One has then:
$$\frac{\cos^2\beta}{\ell_B} - \frac{\cos\beta}{R} + \left(1 - k\frac{\lambda}{\lambda_0}\right)\left(\frac{\cos^2\gamma}{\ell_C} - \frac{\cos\gamma}{R}\right) = 0$$

with
$$\sin\beta + \left(1 - k\frac{\lambda}{\lambda_0}\right)\sin\gamma = 0$$

The polar equation of the tangential focal is thus given by the relation:
$$\frac{\cos^2\beta}{\ell_B} - \frac{\cos\beta}{R} - \frac{\sin\beta}{\sin\gamma}\left(\frac{\cos^2\gamma}{\ell_C} - \frac{\cos\gamma}{R}\right) = 0$$

It's the same as the preceding.

c) - Let's place the source point at point R conjugated from point C so that MR = m MC $\forall M$

Literal expression of Δ is then:

$$\Delta = m\,MC + MB - k\frac{\lambda}{\lambda_0}(MC - MD) - \mathcal{P}$$

thus
$$\Delta = MB + \left(m - k\frac{\lambda}{\lambda_0}\right) MC - k.\frac{\lambda}{\lambda_0}.MD - \mathcal{P}$$

By replacing MA, MC, MD by their limited development, and by equaling at zero the terms in Y and Y^2 one then obtains the system:

$$\begin{cases} \dfrac{\cos^2\beta}{\ell_B} - \dfrac{\cos\beta}{R} + \left(m - k\dfrac{\lambda}{\lambda_0}\right)\left(\dfrac{\cos^2\gamma}{\ell_C} - \dfrac{\cos\gamma}{R}\right) = 0 \\ \sin\beta + \left(m - k\dfrac{\lambda}{\lambda_0}\right)\sin\gamma = 0 \end{cases}$$

The location of the tangential focal is therefore given by the relation:

$$\frac{\cos^2\beta}{\ell_T} - \frac{\cos\beta}{R} - \frac{\sin\beta}{\sin\gamma}\left(\frac{\cos^2\gamma}{\ell_C} - \frac{\cos\gamma}{R}\right) = 0$$

This is the same relation as in the two preceding cases, the proposition is therefore demonstrated.

d) - The proposition remains true if registration points C and D are mixed (equiphased spherical surfaces studied at § 4):

One thus has C and D mixed or: MC = MD

Let's suppose the source point A is located in R conjugated from C, D

or MA = MR = m MC

The expression of Δ is then:

$$\Delta = m\,MC + MC - k\frac{\lambda}{\lambda_0}(2MC) - \rho$$

By placing MC and MB by their limited development and by equalling at zero the terms in Y and Y^2 one obtains the equations:

$$m \sin\gamma + \sin\beta - 2k\frac{\lambda}{\lambda_0}\sin\gamma = 0$$

thus
$$\sin\beta + \left(m - 2k\frac{\lambda}{\lambda_0}\right)\sin\gamma = 0$$

and
$$\frac{\cos^2\beta}{\ell_B} - \frac{\cos\beta}{R} + \left(m - 2k\frac{\lambda}{\lambda_0}\right)\left(\frac{\cos^2\gamma}{\ell_C} - \frac{\cos\gamma}{R}\right) = 0$$

The equation of the tangential focal is therefore always given by the formula:

$$\frac{\cos^2\beta}{\ell_T} - \frac{\cos\beta}{R} - \frac{\sin\beta}{\sin\gamma}\left(\frac{\cos^2\gamma}{\ell_C} - \frac{\cos\gamma}{R}\right) = 0$$

One would demonstrate the proposition in like manner when the source point is in C, D, or at the center of the network.

2°/- Analytical study of the focal curves

- According to the preceding paragraph, it suffices to make the study in a particular case.

One will place source point A at the center of the network: $\alpha = 0 \quad \ell_A = R$

The hypotheses are besides:

- Spherical support : $\lambda = N = 1$

- Point D at the center of the network : $\delta = 0 \quad \ell_D = R$

- By carrying these values in the coefficients of Y, Y^2 and Z^2 one obtains the desired polar equations:

Coefficient of the 1st degree: $\boxed{\sin\beta = k \frac{\lambda}{\lambda_0} \sin\gamma}$

Equation of the sagittal focal:

$$\frac{1}{\ell_S} = \frac{\cos\beta}{R} + \frac{\sin\beta}{\sin\gamma}\left(\frac{1}{\ell_C} - \frac{\cos\gamma}{R}\right)$$

Equation of the tangential focal:

$$\frac{1}{\ell_T} = \frac{\frac{\cos\beta}{R} + \frac{\sin\beta}{\sin\gamma}\left(\frac{\cos^2\gamma}{\ell_C} - \frac{\cos\gamma}{R}\right)}{\cos^2\beta}$$

- A/- <u>The sagittal focal</u>

$$\frac{1}{\ell_S} = \frac{\cos\beta}{R} + \frac{\sin\beta}{\sin\gamma}\left(\frac{1}{\ell_C} - \frac{\cos\gamma}{R}\right)$$

One recognizes the equation of a straight line

It passes by the points $\beta = 0 \quad \ell = R$ or D

and $\beta = \gamma \quad \ell = l_c$ or C

<u>This is therefore the straight line CD</u>.

- B/- <u>The tangential focal</u>:

$$\ell_T = \frac{\cos^2\beta}{\frac{\cos\beta}{R} + \frac{\sin\beta}{\sin\gamma}\left(\frac{\cos^2\gamma}{\ell_C} - \frac{\cos\gamma}{R}\right)}$$

thus

$$\ell_T = \frac{R\cos\beta}{1 + \frac{\operatorname{tg}\beta}{\operatorname{tg}\gamma}\left(\frac{R\cos\gamma}{\ell_C} - 1\right)}$$

propose

$$tg\beta_a = \frac{tg\gamma}{1 - \frac{R\cos\gamma}{\ell_c}}$$

it happens that:

$$\boxed{\ell_T = \frac{R\cos\beta}{1 - \frac{tg\beta}{tg\beta_a}}}$$

- a) - The curve therefore presents an assympototic direction for the polar angle β_a so that:

$$tg\beta_a = \frac{tg\gamma}{1 - \frac{R\cos\gamma}{\ell_c}}$$

By convention one takes: $0 \leq \gamma \leq \frac{\pi}{2}$

If $\ell_c > R\cos\gamma$ therefore if point C is exterior to the circle of Rowland: $\beta_a > 0$

If $\ell_c < R\cos\gamma$ therefore if the point C is interior to the circle of Rowland: $\beta_a < 0$

The position of the assymptote is defined by the quantity

$$d = \overline{IH} = \text{limit of } \lim_{\beta \to \beta_a} \ell_T . \sin(\beta - \beta_a) \quad \text{(see figure 12)}$$

But one can write ℓ_T in the form:

$$\ell_T = \frac{R\cos\beta . \sin\beta_a}{\sin(\beta_a - \beta)}$$

whence

$$\boxed{d = -R(\sin\beta_a - \sin^3\beta_a)}$$

For not too important angles Ba $\sin^3 Ba \ll \sin Ba$

therefore: $d \propto -R \sin \beta a$

The assymptote passes in the neighborhood of the center of the network:

- b) - For $\beta = \pm \frac{\pi}{2}$ on a : $\ell_T = 0$

The focal curve thus passes by the summit of the network with a vertical tangent.

- c) The intersection points of the focal tangential curve with the sagittal curve are obtained by writing that one has (unreadable) ℓ_T

- One already knows that these two curves meet at least in three points: the points of harsh stigmatism.

The resolution of the equation $\ell_S - \ell_T = 0$ furnished 3 values of β, they are therefore necessarily the 3 stigmatic points:

thus $\beta 1 = 0 \quad \ell = R \rightarrow$ center of the network

$\beta_2 = \gamma \quad \ell = \ell_c \rightarrow$ Point C

β_3 with $tg\beta_3 = tg\gamma \dfrac{\frac{R\cos\gamma}{\ell_c}}{1 - \frac{R\cos\gamma}{\ell_c}}$

- β_3 is therefore the polar angle of point R conjugated harmonic of point C

$$tg\beta_3 = \frac{tg\gamma}{1 - \frac{R\cos\gamma}{\ell_c}} \cdot \frac{R\cos\gamma}{\ell_c} = tg\beta a \cdot \frac{R\cos\gamma}{\ell_c}$$

$\beta a > \beta_3$ if C is exterior to the circle of Rowland

$\beta a < \beta_3$ if C is interior to the circle of Rowland

- One notices that for $\ell_c = \infty \quad \beta_3 = 0 = \beta_1$

$\beta = 0$ is root double the tangential curve, it is therefore tangent to the straight line location of the sagittal focal at the center of the network, the astigmatism therefore remains almost nil in the neighborhood of this point.

- For $\ell_c = 2R\cos\gamma$, therefore when point C is located on the circle of the network, $\beta_3 = \gamma = \beta_2$

The sagittal and tangential focal curves are therefore tangent to point C, the astigmatism remains therefore also nearly nil in the neighborhood of C for this configuration.

The derivative of ℓ_T with relation to β is not studied simply, it has as its expression

$$\frac{d\ell_T}{d\beta} = R \cdot \frac{1 - \sin\beta\cos\beta\,(tg\beta_a - tg\beta)}{\cos\beta \cdot tg\beta_a\left(1 - \frac{tg\beta}{tg\beta_a}\right)}$$

- The angle $\hat{V}$ between the tangents in a point of the focal tangential curve and the vector radius is given by the expression:

$$tg\,V = \frac{\ell_T}{\frac{d\ell_T}{d\beta}}$$

thus:

$$tg\,V = \frac{\cos\beta\,(tg\beta_a - tg\beta)}{1 - \sin\beta\cos\beta\,(tg\beta_a - tg\beta)}$$

- Figure 13 gives the direction of the focal curve when point C is at the exterior of the circle of Rowland of the network.

- Figure 14 gives the direction of the focal curve when C is at the interior of the circle of Rowland. One establishes that in that case the third point of stigmatism is quite potential.

- Figure 15 gives the evolution of the focal curve as a function of the step of the network for a same value of m, therefore the same third wave length of stigmatism: λ_H = 6000 (Point C exterior to the circle of Rowland).

- Figure 16 gives the evolution of the focal curves as a function of m for a given step: 1200 lines/mm.

- λ correction = m 4880 A^o

$$OC = \frac{R}{m}$$

OR = m.R.

Number of lines/mm	γ
600	17^o
1200	36^o
1800	$61^o.5$
2000	90^o

Value of m	λ correction A^o
2.05	10,000
1.43	7,000
1.23	6,000
1	4,880
0.82	4,000
0.51	2,500
0.246	1,200
0.123	600

The case m = o where point C is rejected at   is equally interesting.

Examination of the sagittal and tangential focal curves show that these curves are in general secant to the stigmatic points, the stigmatism therefore decreases rather quickly in the neighborhood of these points.

- These curves are found to be tangent only in two cases, when point C is at infinity and when it is on the circle of the network: we are going to study in those cases in the neighborhood of the point of tangence, how the Coma varies:

1°/- Registration point C is at infinity in the direction γ : on the other hand, the other registration point D is at the center of the network, source point A is at infinity in the same direction as C, and one observes in the neighborhood of the center of the network

One has therefore $\left[\begin{array}{ll} \ell_C = \infty & \\ \ell_D = R & \delta = 0 \\ \ell_A = \infty & \alpha = \gamma \end{array}\right.$

The expression of the aberrant deviations therefore becomes:

For tangential astigmatism $$\Delta_T = \frac{Y^2}{2}\left[\frac{\cos^2\beta}{\ell_B} - \frac{\cos\beta}{R} + \frac{\cos\gamma}{R}\cdot\frac{\sin\beta}{\sin\gamma}\right]$$

For sagittal astigmatism $$\Delta_S = \frac{Z^2}{2}\left[\frac{1}{\ell_B} - \frac{\cos\beta}{R} + \frac{\cos\gamma}{R}\cdot\frac{\sin\beta}{\sin\gamma}\right]$$

For Coma $$\Delta_c = \frac{Y^3}{2}\left[\frac{\sin\beta}{\ell_B}\left(\frac{\cos^2\beta}{\ell_B} - \frac{\cos\beta}{R}\right)\right]$$

One does have $\Delta_T = \Delta_S = \Delta_c = 0$, for d_0 , at the center of the network and one knows that the astigmatism remains nil in the neighborhood, let's calculate $\dfrac{d\Delta_c}{d\beta}$

$$\frac{d\Delta_c}{d\beta} = \frac{Y^3}{2}\left[\frac{1}{\ell_B}\left[\cos^2\beta\left(\frac{\cos\beta}{\ell_B}-\frac{1}{R}\right)+\sin^2\beta\left(\frac{1}{R}-\frac{2\cos\beta}{\ell_B}\right)\right]+\right.$$
$$\left.+\,2\,\frac{\sin\beta\cos^2\beta}{\ell_B}\cdot\frac{d\left(\frac{1}{\ell_B}\right)}{d\beta}-\frac{\sin\beta.\cos\beta}{R}\cdot\frac{d\left(\frac{1}{\ell_B}\right)}{d\beta}\right]$$

for $\beta = o$ $\quad \ell_\beta = R$ one has therefore $\frac{d\Delta_c}{d\beta} = o$

The coma remains therefore also very weak in the neighborhood of the center.

- It is necessary however to notice that this last property A' is true only when points A and C are mixed at infinity, that is, for the wave length 4880 at the center of the network.

- One can try to evaluate the height of the tangential focal:

If H is the height of the line of the network one has:

$$h_T = H\,\frac{\ell_S - \ell_T}{\ell_S}$$

Thus here $\boxed{hT = H.\sin^2\beta}$

This expression remains true if point A is not mixed with point C at infinity.

- One can therefore compare this mounting to the standard mounting of Wadworth.

In this case:

$$\Delta_T = \frac{Y^2}{2}\left[\frac{\cos^2\beta}{\ell_B}-\frac{\cos\beta}{R}-\frac{\cos\gamma}{R}\right]$$

$$\Delta_S = \frac{z^2}{2}\left[\frac{1}{\ell_B}-\frac{\cos\beta}{R}-\frac{\cos\gamma}{R}\right]$$

$$\Delta_C = \frac{Y^3}{2}\left[\frac{\sin\beta}{\ell_B}\left(\frac{\cos^2\beta}{\ell_B}-\frac{\cos\beta}{R}\right)\right]$$

One has also $hT = H \sin^2 \beta$

But one never has $\frac{d \Delta c}{d \beta} = 0$ for $\beta = 0$

- Figure 17 permits comparing the arrangement of the two mounting.

- The linear dispersion $\frac{d\ell}{d\lambda}$ is multiplied by $\frac{1}{\sin \gamma}$ for the holographic mounting.

2°)- Registration point C is on the circle of the network.

- The other registration point D is at the center of the network as well as source point A.

- One observes in the neighborhood of point C

One has therefore
$$\begin{cases} \ell_c = 2R\cos\gamma \\ \ell_A = \ell_D = R \\ \alpha = \delta = 0 \end{cases}$$

The epxression of the Coma is then:

$$\Delta_c = \frac{Y^3}{2}\left[\frac{\sin\beta}{\ell_B}\left(\frac{\cos^2\beta}{\ell_B} - \frac{\cos\beta}{R}\right) - \frac{\sin\beta}{\ell_c}\left(\frac{\cos^2\gamma}{\ell_c} - \frac{\cos\gamma}{R}\right)\right]$$

by replacing ℓ_c by its value:

$$\Delta_c = \frac{Y^3}{2}\left[\frac{\sin\beta}{\ell_B}\left(\frac{\cos^2\beta}{\ell_B} - \frac{\cos\beta}{R}\right) + \frac{\sin\beta}{4R^2}\right]$$

Then one has as an expression of the derivative of Δ_c in relation to β

$$\frac{d\Delta_c}{d\beta} = \frac{Y^3}{2}\left[\frac{1}{\ell_B}\left[\cos^2\beta\left(\frac{\cos\beta}{\ell_B} - \frac{1}{R}\right) + \sin^2\beta\left(\frac{1}{R} - \frac{2\cos\beta}{R}\right)\right] + 2\,\frac{\sin\beta\cos^2\beta}{\ell_B}\cdot\frac{d\frac{1}{\ell_B}}{d\beta} - \frac{\sin\beta\cos\beta}{R}\cdot\frac{d\frac{1}{\ell_B}}{d\beta} + \frac{\cos\beta}{4R^2}\right]$$

for $\ell_B = 2R\cos\beta$ one has therefore $\frac{d\Delta_c}{d\beta} = 0$

- In the neighborhood of the stigmatic point C, the astigmatism and the coma remain therefore very weak.

8^{o}/- Application to spectroscopy:

I - Aberration introduced by the height of the slots.

- The general expression which gives the aberrant deviation Δ has been established for points A, B, C, D all located in the plane XY.

- To take into account the finite height of the slots of a spectrograph utilizing the network, one is led to give coordinates at these different points.

- However, to conserve the expressions already established, we shall adopt a system of cylindrical coordinates of axis $\overrightarrow{IZ}$.

The coordinates of A, B, C, D, will then be : (see Figure 18)

$$A : \begin{bmatrix} \alpha \\ \ell_A \\ z_A \end{bmatrix} \quad \text{with} \quad \begin{matrix} x_A = \ell_A \cos\alpha. \\ y_A = \ell_A \sin\alpha \\ z_A . \end{matrix}$$

$$B : \begin{bmatrix} \beta \\ \ell_B \\ z_B \end{bmatrix} . \quad \text{with} \quad \begin{matrix} x_B = \ell_B \cos\beta \\ y_B = \ell_B \sin\beta \\ z_B \end{matrix}$$

$$C : \begin{bmatrix} \gamma \\ \ell_C \\ z_C \end{bmatrix} \quad \text{with} \quad \begin{matrix} x_C = \ell_C \cos\gamma \\ y_C = \ell_C \sin\gamma \\ z_C \end{matrix}$$

$$D : \begin{bmatrix} \delta \\ \ell_D \\ z_D \end{bmatrix} \quad \text{with} \quad \begin{matrix} x_D = \ell_D \cos\gamma \\ y_D = \ell_D \sin\gamma \\ z_D . \end{matrix}$$

- One will try to obtain a limited development of quantities of type MA, in relation to the variables of the first order Y, Z and also γ

- As in § one will be able to put MA in the form

$$MA = \ell\left(1 + \frac{\theta}{2} - \frac{\theta^2}{8} + \frac{\theta^3}{16} + \ldots\ldots\right)$$

with

$$\theta = \frac{\zeta^2}{\ell^2} - 2\,\frac{yY+\zeta Z}{\ell^2} + \frac{Y^2+Z^2}{\ell^2} - \frac{x}{R\ell^2}\left(\lambda Y^2+\mu Z^2\right) + \left(\frac{\lambda Y^2+\mu Z^2}{\ell^2}\right)^2\left(\frac{1}{4R^2} - \frac{\lambda x}{4R^3}\right)$$

By arranging according to increasing powers, Y, Z, ζ one will obtain as an expression of MA:

MA = ℓ

1st order $\quad - Y \sin\alpha$

2nd order $\quad + \dfrac{\zeta^2}{2\ell} - Z\dfrac{\zeta}{\ell} + \dfrac{Y^2}{2}\left(\dfrac{\cos^2\alpha}{\ell} - \dfrac{\lambda\cos\alpha}{R}\right) + \dfrac{Z^2}{2}\left(\dfrac{1}{\ell} - \mu\dfrac{\cos\alpha}{R}\right)$

3rd order

$$\left[\begin{array}{l} + \dfrac{Y}{2}\sin\alpha\,\dfrac{\zeta^2}{\ell^2} - YZ\,\dfrac{\zeta}{\ell^2}\sin\alpha \\ + \dfrac{Y^3}{2}\,\dfrac{\sin\alpha}{\ell}\left(\dfrac{\cos^2\alpha}{\ell} - \dfrac{\lambda\cos\alpha}{R}\right) \\ + \dfrac{YZ^2}{2}\,\dfrac{\sin\alpha}{\ell}\left(\dfrac{1}{\ell} - \mu\dfrac{\cos\alpha}{R}\right) \end{array}\right.$$

+ 4 in order

- By carrying this expression in the literal expression of Δ (5) one obtains as aberrant <u>complementary</u> terms:

In the second order: $-Z\left[\dfrac{\zeta_A}{\ell_A} + \dfrac{\zeta_B}{\ell_B} - k\dfrac{1}{d_0}\left(\dfrac{\zeta_C}{\ell_C} - \dfrac{\zeta_D}{\ell_D}\right)\right]$

In the third order: $+\dfrac{Y}{2}\left[\sin\alpha\,\dfrac{\zeta_A^2}{\ell_A^2} + \sin\beta\,\dfrac{\zeta_B^2}{\ell_B^2} - k\dfrac{1}{d_0}\left(\sin\gamma\,\dfrac{\zeta_C^2}{\ell_C^2} - \sin\delta\,\dfrac{\zeta_D^2}{\ell_D^2}\right)\right]$

$$-YZ\left[\sin\alpha\,\frac{\zeta_A}{\ell_A^2} + \sin\beta\,\frac{\zeta_B}{\ell_B^2} - k\frac{1}{d_0}\left(\sin\gamma\,\frac{\zeta_C}{\ell_C^2} - \sin\delta\,\frac{\zeta_D}{\ell_D^2}\right)\right]$$

- One will take in general $\zeta_C = \zeta_D = 0$ one then has as aberrant <u>complementary</u> terms:

A) - In the second order:

$$\Delta' = -Z\left(\frac{\gamma_A}{\ell_A} + \frac{\gamma_B}{\ell_B}\right)$$

One can nullify it by making:

$$\boxed{\frac{\gamma_A}{\ell_A} = -\frac{\gamma_B}{\ell_B}}$$

which corresponds to a relation of geometric increase.

B) - In the third order:

Taking into account the preceding relation:

$$\Delta' = \frac{Y}{2}\frac{\gamma_B^2}{\ell_B^2}\left(\sin\alpha + \sin\beta\right) = \frac{Y}{2}\frac{\gamma_A^2}{\ell_A^2}\left(\sin\alpha + \sin\beta\right)$$

- which corresponds to a distortion of the picture.

- the latter becomes a curve with a radius R_c:

$$R_c = \frac{\ell}{\sin\alpha} \qquad \text{in normal diffraction}$$

$$Rc = \frac{\ell}{2\sin\alpha} \qquad \text{in autocollination}$$

C) - In the third order:

$$\Delta' = -YZ\left(\sin\alpha\,\frac{\gamma_A}{\ell_A^2} + \sin\beta\,\frac{\gamma_B}{\ell_B^2}\right)$$

Which corresponds to an enlargement of the picture.

Littrow order I 1

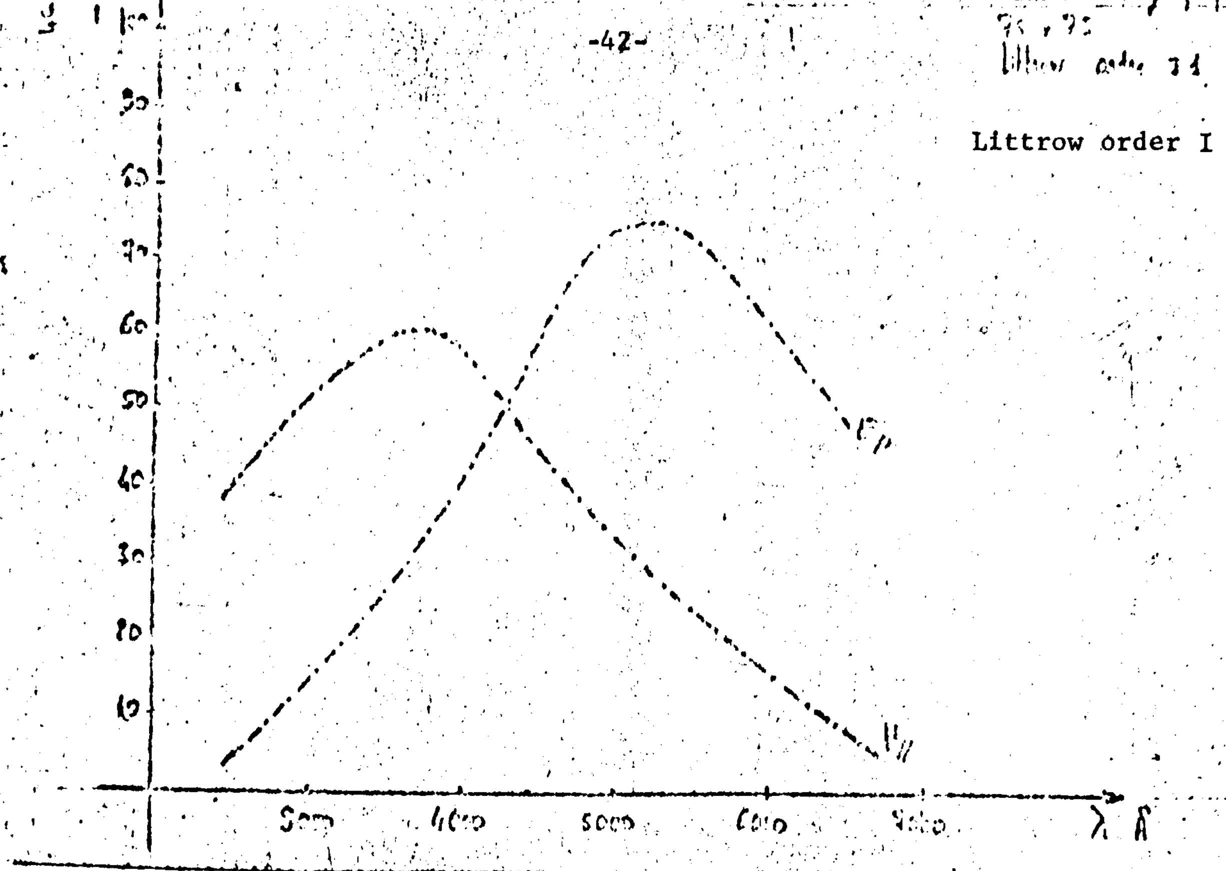


Fig I.

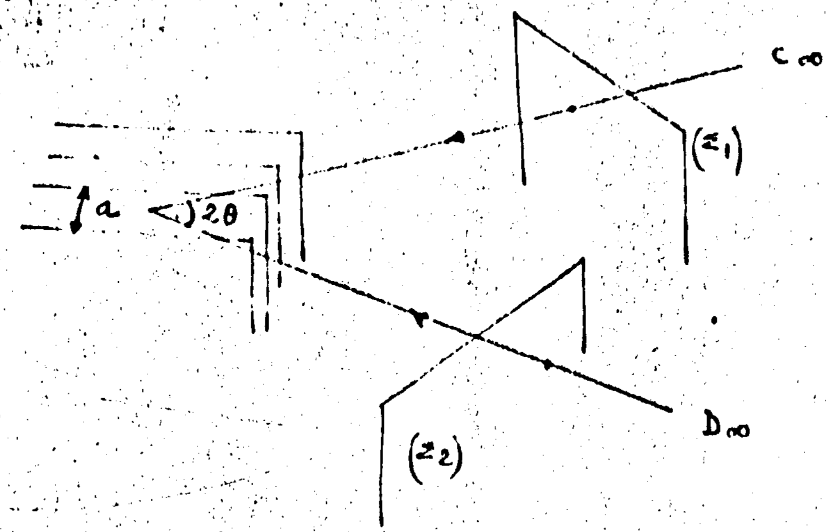


Fig II

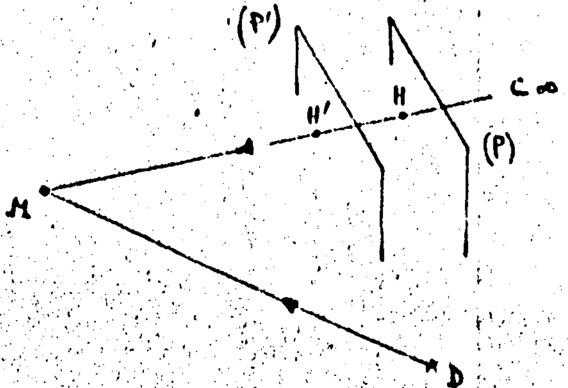


Fig III

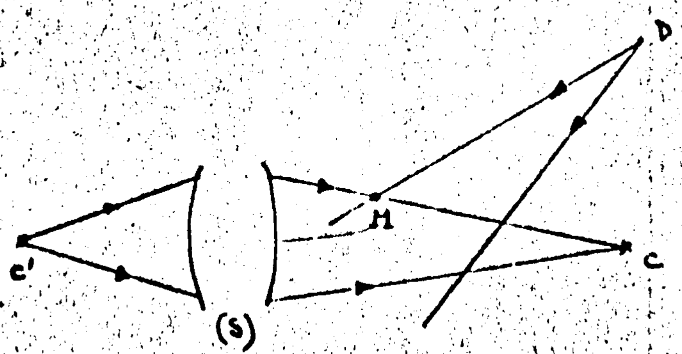


Fig IV

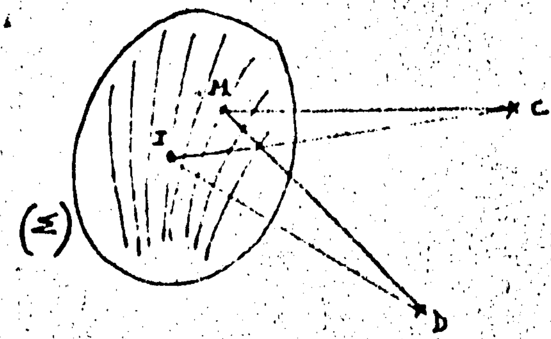


Fig V

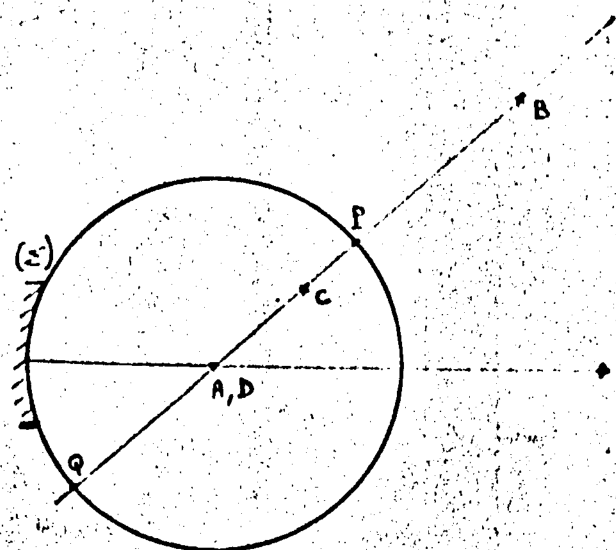
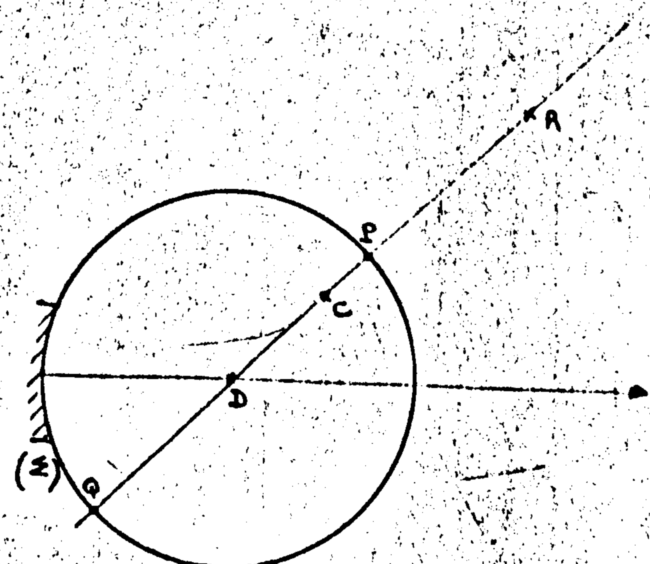
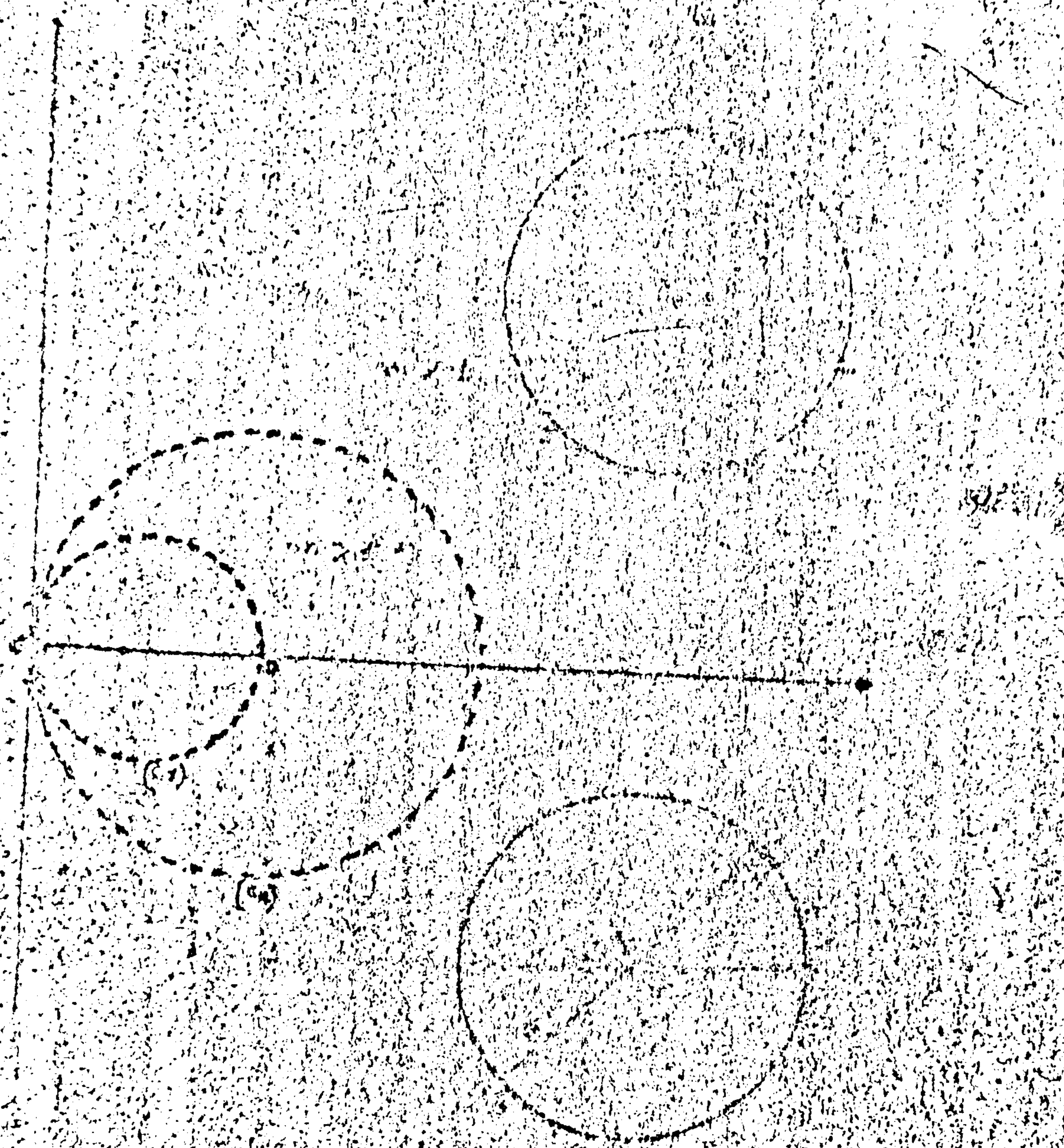


Fig VI





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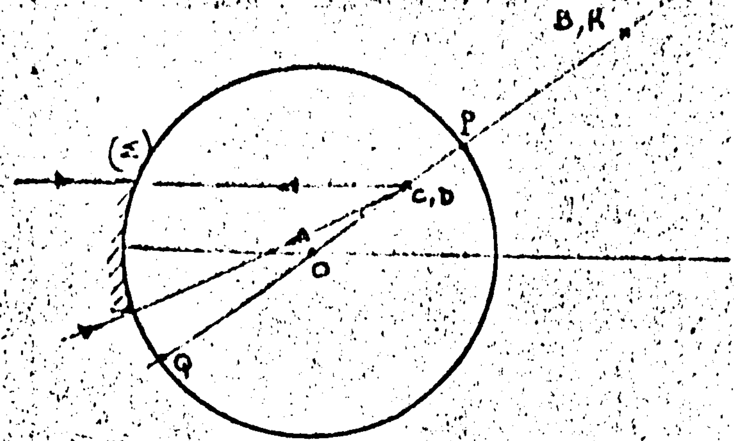


Fig VIII

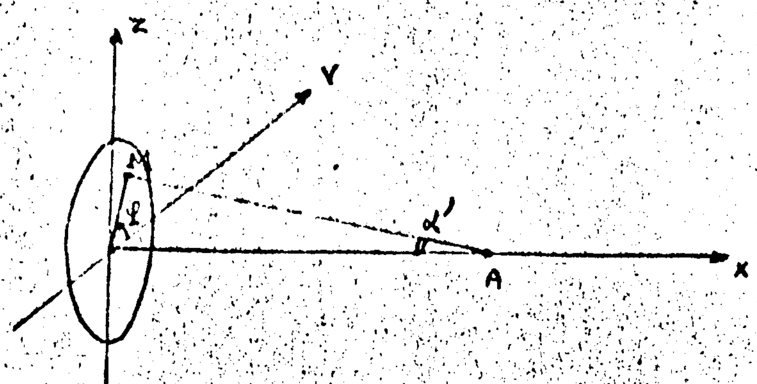


Fig IX

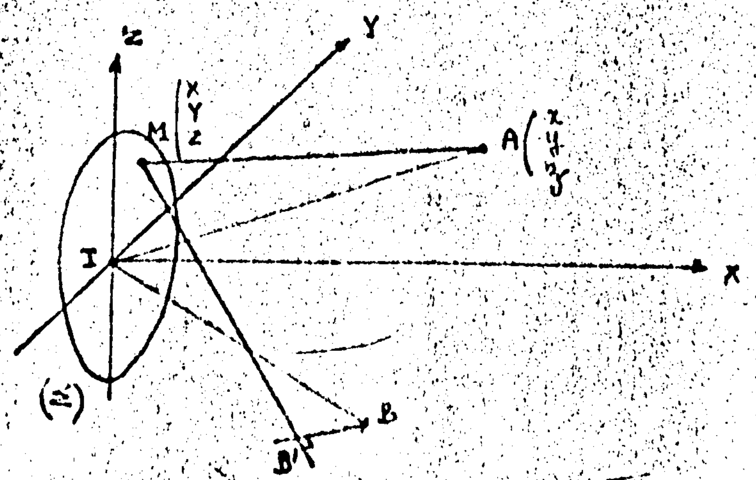
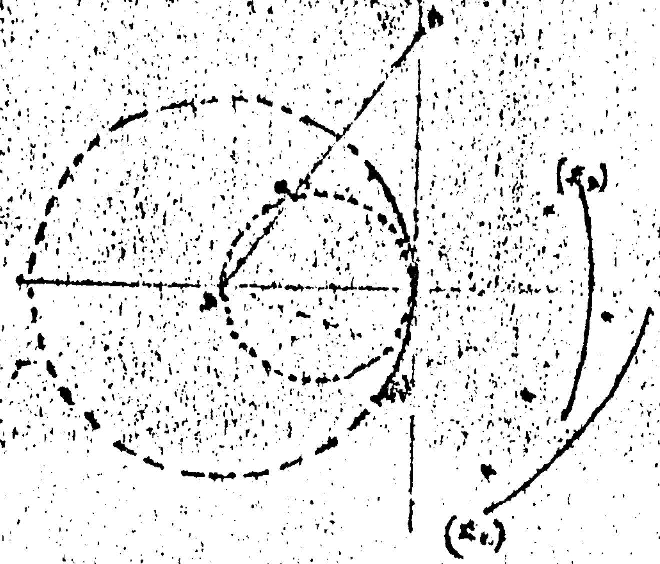
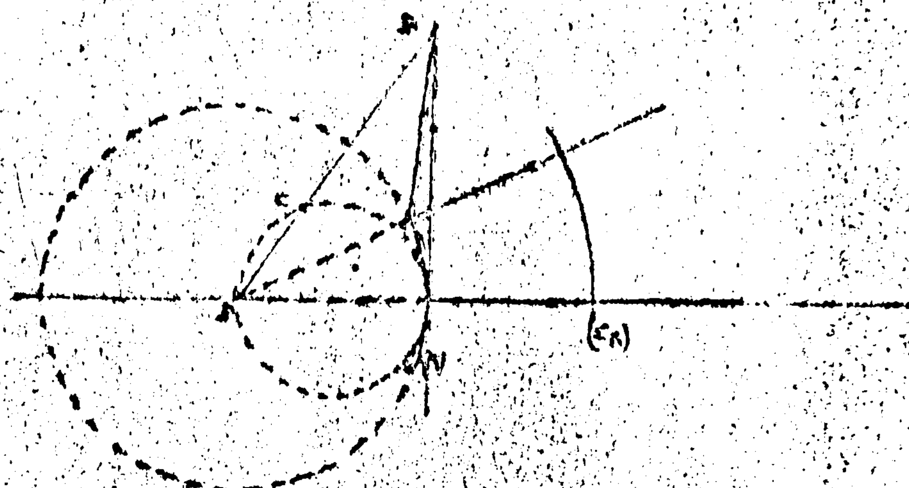
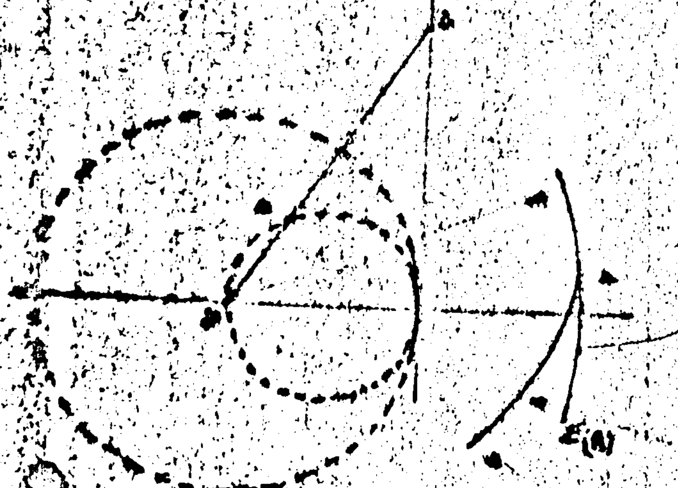
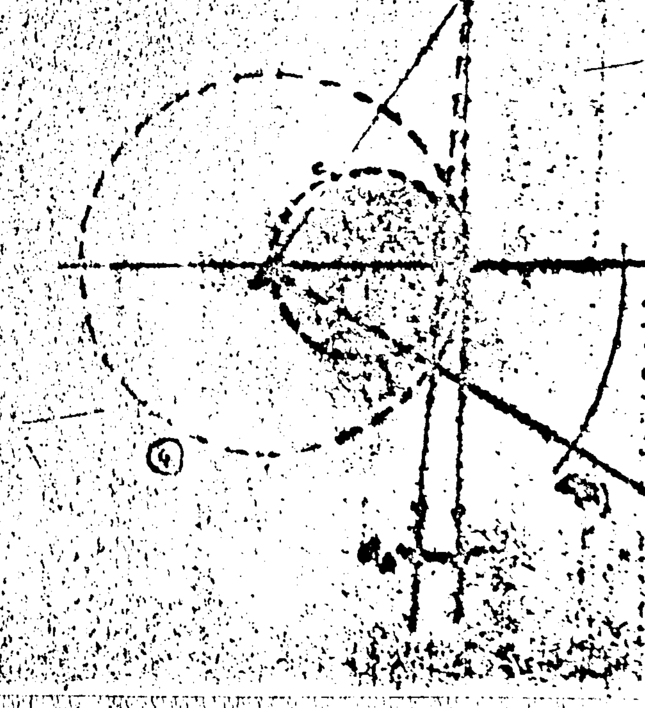


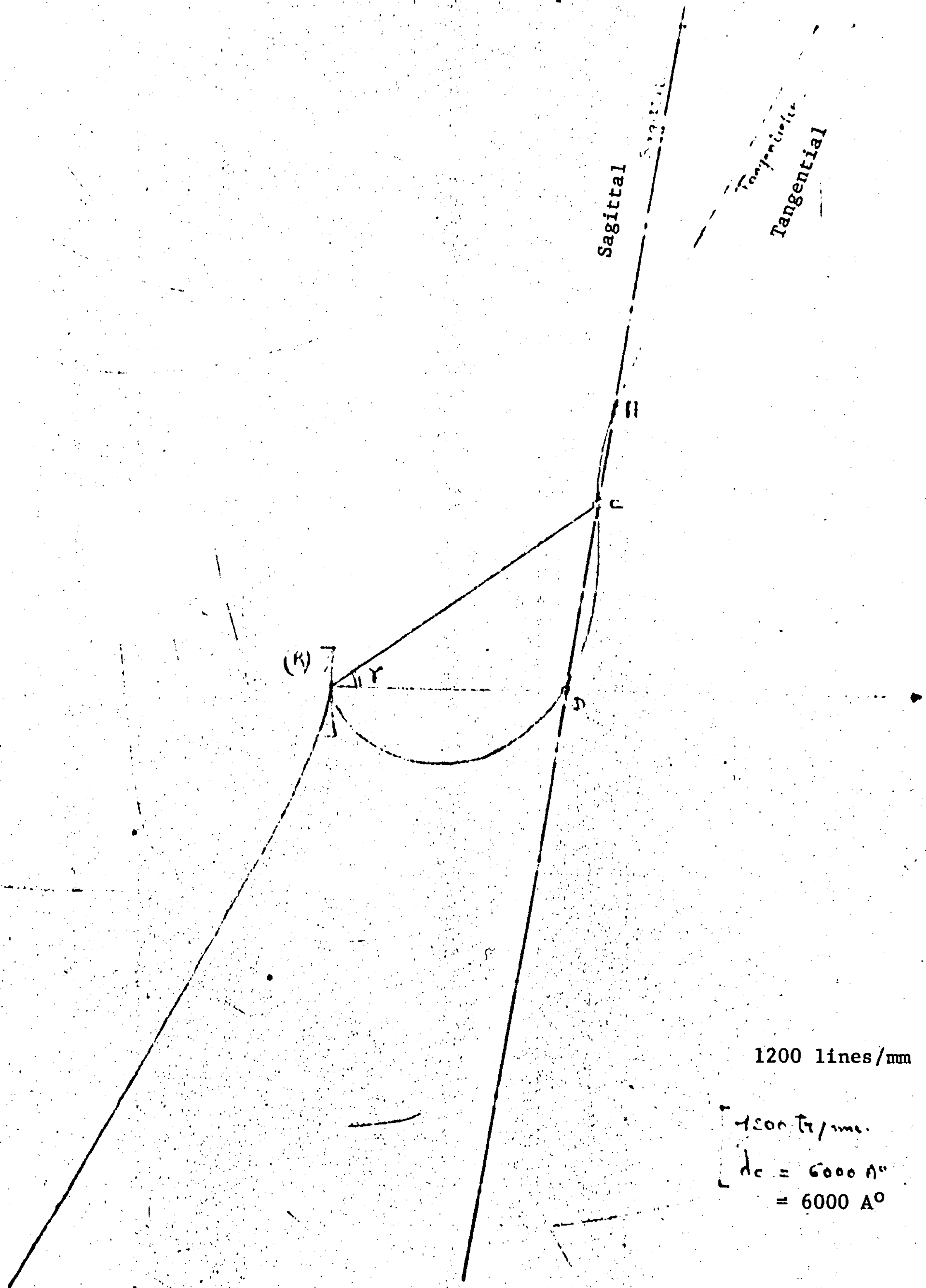
Fig X





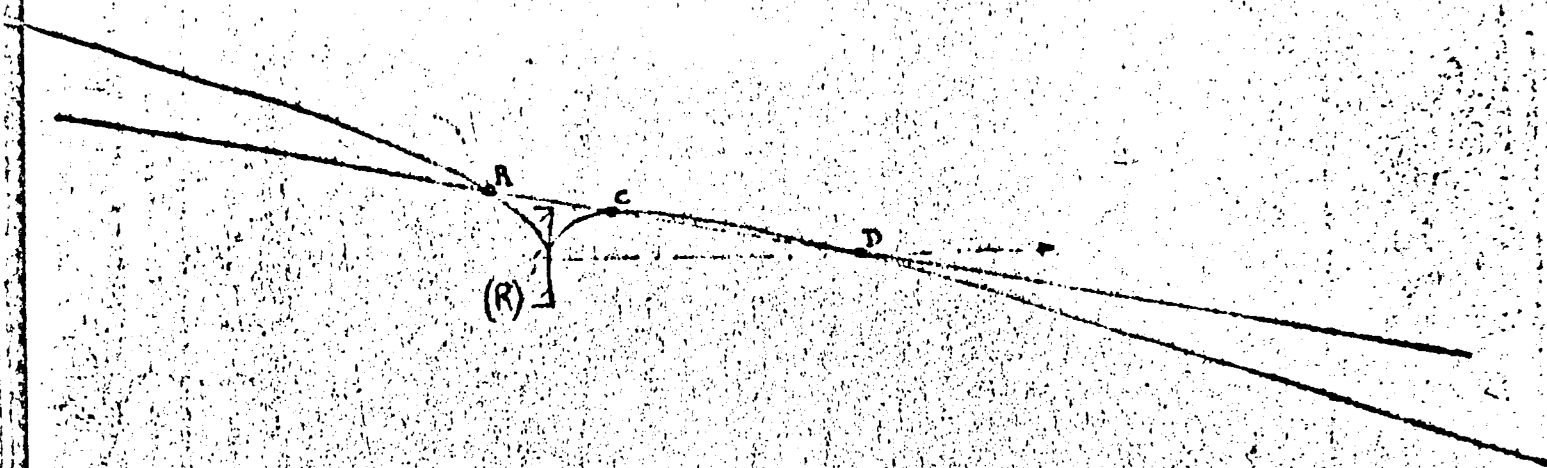






Focal curves for c at the exterior of the circle of Rowland

Fig XIII courbes Focales pour c à l'extérieur du cercle de Rowland

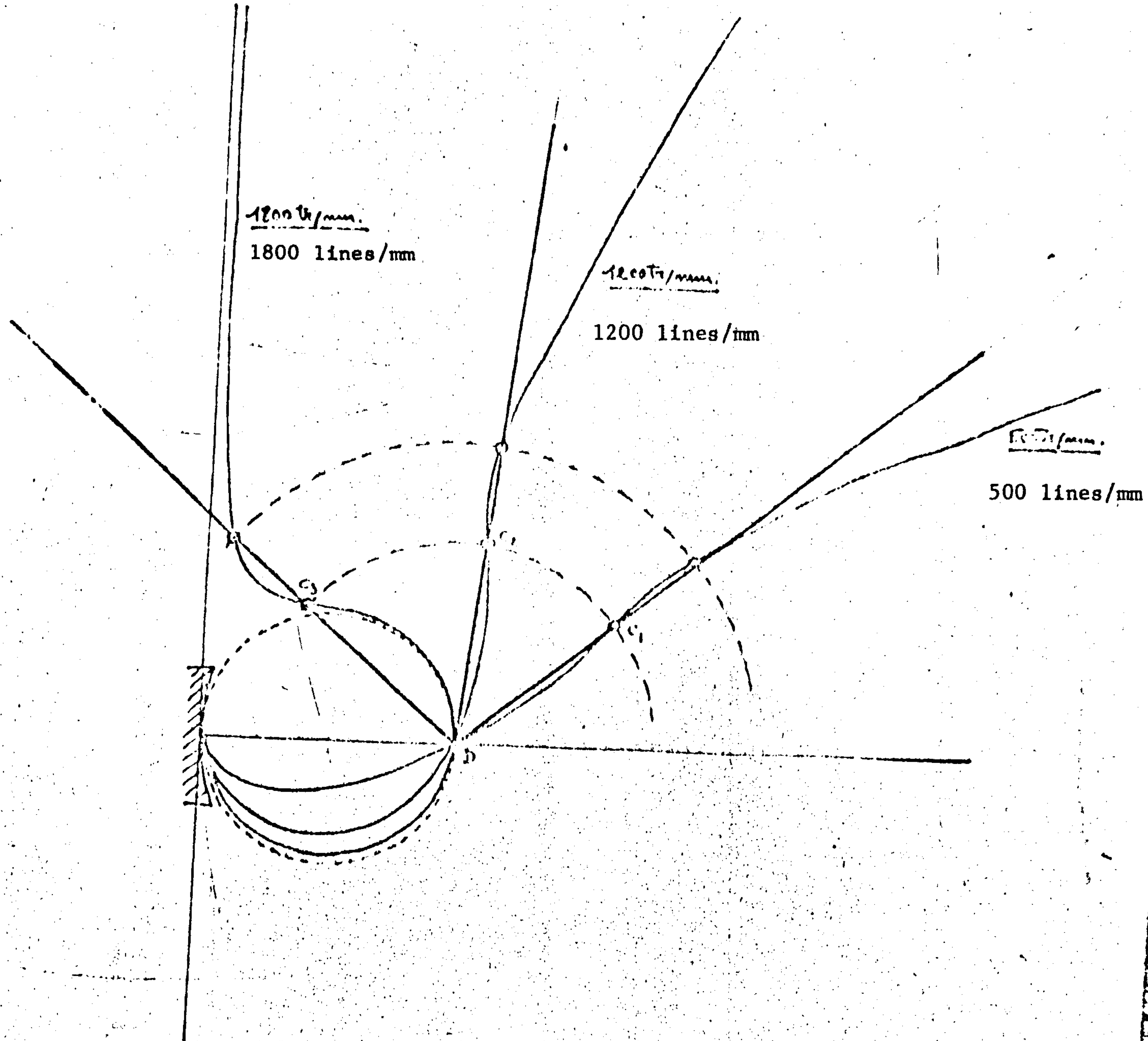


1200 lines/mm

1200 tr/mm
λ_c = 6000 A°

Focal curves for c at the interior of the circle of Rowland

Fig XIV. courbes focales pour c à l'intérieur du cercle de Rowland



Focale tangentielle . Tangential focal

Focale sagittale Sagittal focal

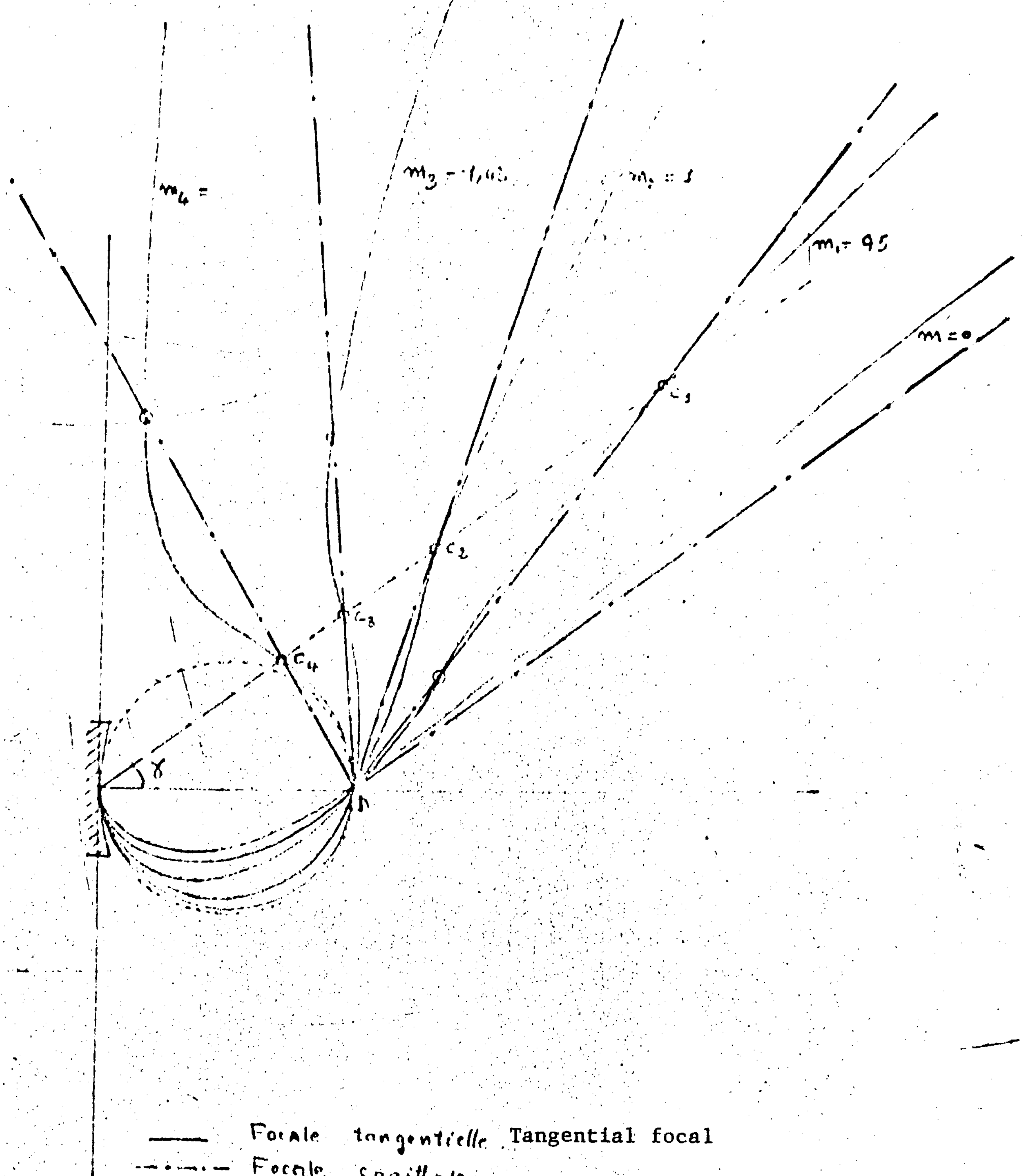
Evolution des courbes tangentielles avec le pas

Evolution of the tangential curves with the step

m = 1,23 fini → λc = 6000 A°

m = 1.23 finite →

Fig 15



—— Focale tangentielle Tangential focal

—·—·— Focale sagittale Sagittal focal

Evolution des courbes tangentielles avec m

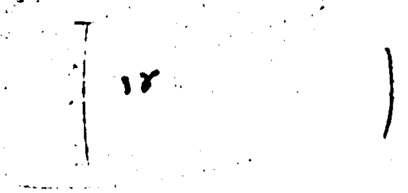
Evolutions of the tangential curve with m

m = dc / do

Pas fixe = 1200 tr/min.

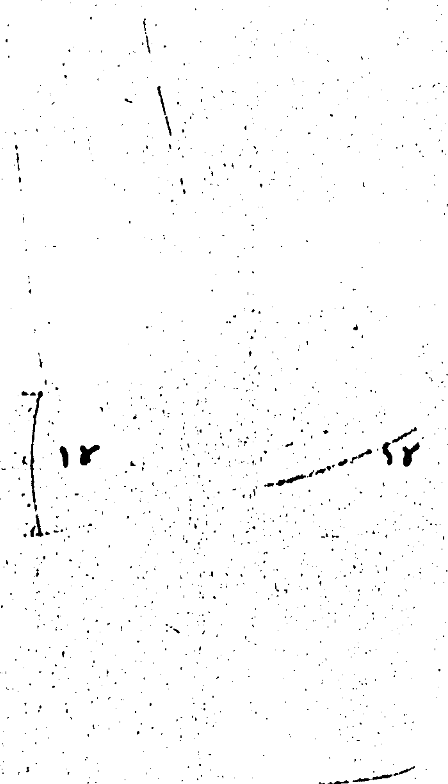
Fixed step =

Fig 16



- Montage Wadworth

- Wadworth mounting



- Montage holographique.

Holographic mounting

-.fig 17.-

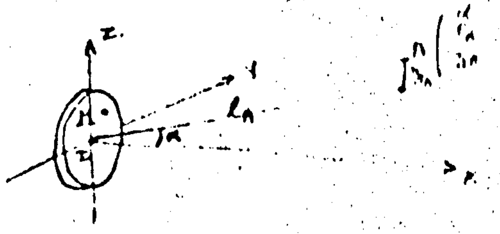


fig 18.

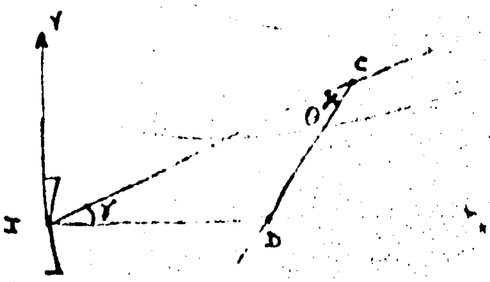
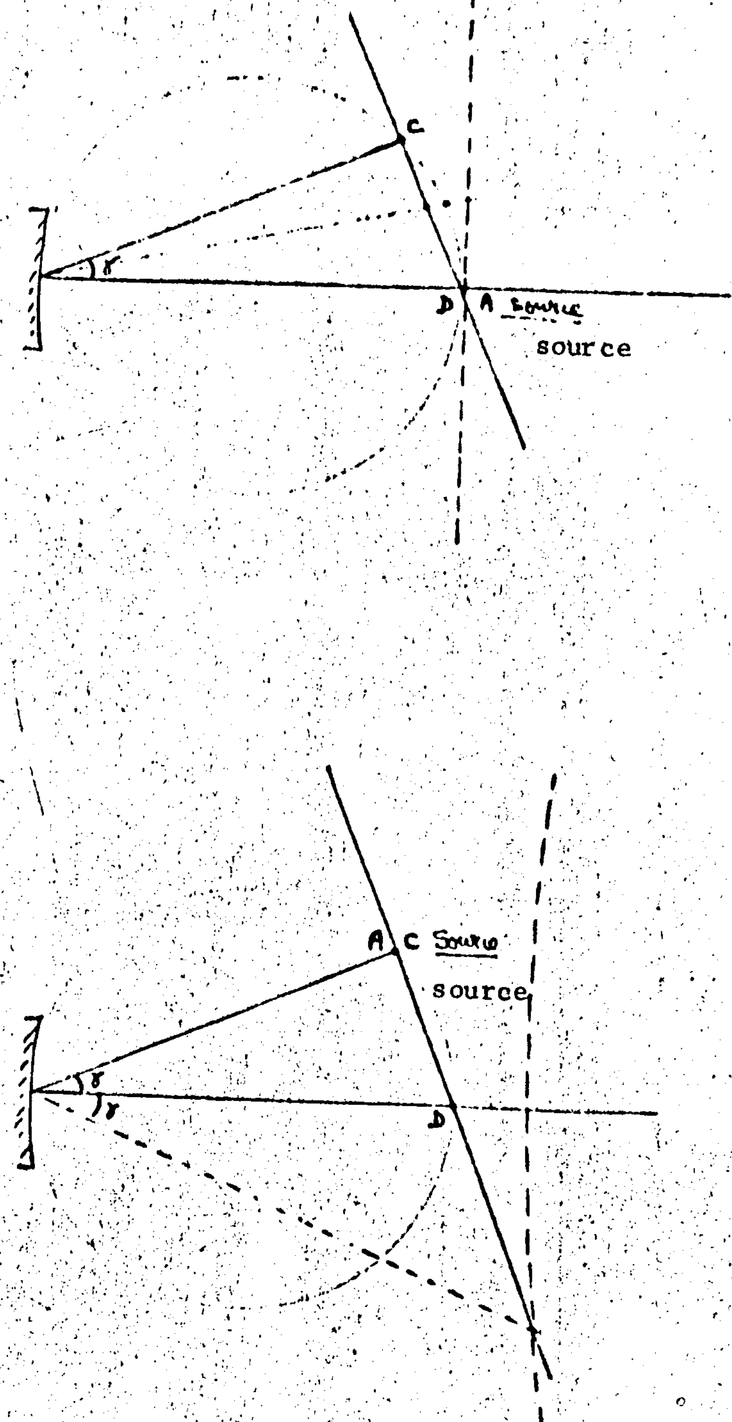


fig 19.



Location of the sagittal focal for the holographic network.

Lieu de la focale sagittale pour le réseau holographique

Location of the sagittal focal for the standard network.

Lieu de la focale sagittale pour le réseau classique.

Circle of Rowland, location of the tangential focal.

Cercle de Rowland lieu de la focale tangentielle.

— fig 20 —